LECTURES ON LOGARITHMIC ALGEBRAIC GEOMETRY

by Lorenzo Ramero

Talk 1 : **Toric varieties**

We introduce toric varieties and toroidal embeddings, following Mumford's original presentation in [1]. We explain (without proofs) how the formalism of toric varieties translates algebraic geometric questions into combinatorial questions concerning certain monoids. We also discuss how one solves some of these combinatorial problems via the theory of convex polyhedral cones.

Talk 2 : **Log schemes**

We explain how the theory of toric varieties motivated the definition of "log structure" on a scheme. We introduce the basic notions of the theory : local charts of a log structure, fine and saturated log schemes, sheaf of logarithmic differentials, étale and smooth morphisms of log schemes, and their local structure. Our reference is chapters 3 and 4 of the treatise [2].

Talk 3 : **Regular log schemes and logarithmic Abhyankar's lemma**

We explain the logarithmic counterpart of the classical notion of regularity for schemes. In the case where the base scheme is a field, the resulting class of regular log schemes is none else than Mumford's class of toroidal embeddings. In the case where the base is a DVR, one still obtains a good local description, similar to Cohen's structure theorem for complete regular local rings.

We state the logarithmic Abhyankar's lemma for regular log schemes, that identifies the étale coverings of a given regular log scheme, with the finite coverings of the underlying log scheme that are tamely ramified along the locus of non-triviality of the log structure. If time allows, we shall sketch a proof of this theorem, and derive some corollaries, with examples. The reference is chapter 5 of [2].

REFERENCES

[1]   D.MUMFORD ET AL. "Toroidal embeddings. I", Springer LNM 339 (1973).

[2]   O.GABBER AND L.RAMERO. "Foundations of p-adic Hodge theory", Third release. Preprint available on http://math.univ-lille1.fr/~ramero and on the math arXiv.