**Monodromy and Hypergeometric Functions**

17-21.02.2020, Galatasary University, Istanbul, Turkey

Irina Antipova (SFU, Krasnoyarsk)

Title: Analytic continuations of algebraic functions by means of Mellin--Barnes integrals

Abstract: In my talk, I will speak about the techniques of analytic implementation of the monodromy of the algebraic function given by the system of algebraic equations. The tool of the analytic extension is the Mellin-Barnes integral representation for the monomial function of coordinates of the solution to the system. In 1921 Hjalmar Mellin proposed to solve algebraic equations by means of hypergeometric series and integrals [Me]. For the last two decades, this approach was developed in a series of papers (see, for instance, [A07], [St03], [AMi]). While a power series of the solution converges in a polycircular domain, a Mellin-Barnes integral converges in a sectorial domain given by conditions for arguments of variables. Since the intersection of these domains is always nonempty, the series expansion of the solution admits an analytic continuation into the sectorial domain by means of the integral. The calculation techniques for the Mellin-Barnes integrals is based on the separating cycle principle formulated in [Ts92].

References:

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[A07] Antipova, I.A.: Inversion of many-dimensional Mellin transforms and solutions of algebraic equations. Sb. Math. 198 :4, 447-463 (2007)

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[AMi] Antipova, I.A., Mikhalkin, E.N.: Analytic continuations of a general algebraic function by means of Puiseux series. Proc. Steklov Inst. Math. \textbf{279}, 3-13 (2012)

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Frits Beukers (Utrecht)

Title: The invariant Hermitian form for the monodromy of A-hypergeometric systems (after C.Verschoor).

Abstract: In this lecture we will explain how to compute the invariant Hermitian form for the monodromy of A-hypergeometric systems with real parameters. Our aim is to do this without using intersection theory of twisted cycles. This can be achieved for a limited set of systems. We add some speculations for the general cases.

Jens Forsgård (Utrecht)

Title: Computing generators of fundamental groups using amoebas

Abstract: We discuss the problem of determining a generating set of the fundamental group of the complement of a discriminantal hypersurface using amoebas (i.e., projections of algebraic varieties under multiplicative morphisms).  In particular, we give conditions under which Beuker's local monodromy group of the A-hypergeometric system coincides with the full monodromy group.

Yoshiaki Goto (Otaru)

Title: Finite irreducible monodromy group for Lauricella's F\_C

Abstract: We consider the monodromy group for Lauricella's hypergeometric function F\_C. In this talk, I would like to give the conditions under which the monodromy group is finite irreducible. It is a generalization of the finiteness conditions of the monodromy group for Appell's F\_4, which was given by M. Kato (1997). I would also like to explain the structure of the fundamental group of the complement of the singular locus.

Paul R. Horja (Miami)

Title: Toric Schobers and D-modules

Abstract: Many classical mirror symmetry results can be recast using the recent language of perverse sheaves of categories and schobers. In this context, I will explain a Riemann-Hilbert type conjectural connection with the D-modules naturally appearing in mirror symmetry. This is joint work with Ludmil Katzarkov.

Alexei Ivanov (HSE, Moscow)

Title: Feynman integrals and mirror symmetry

Abstract:

In this talk I will review the recent results concerning the geometric nature of the Feynman integrals. More precisely, a Feynman integral corresponding to a Feynman diagram can be always represented as the integral of a rational differential form over some non-closed cycle in the projective space. It immediately leads to the representation of the Feynman integrals as the periods of the mixed Hodge structures. Based on the works by P. Vanhove, S. Bloch, M. Kerr (and others), we will discuss in detail the case of the sunset Feynman diagram which is associated to the Calabi-Yau geometry and has very intriguing link with the mirror symmetry. Also we will try to understand how to generalize this approach to the case of more difficult Feynman diagrams like 2-loop ladder diagram for which almost nothing is known.

Michael Lönne (Bayreuth)

Title:  On the topology of complements of hyperplane arrangements and some of their quotients

Abstract: Given a hyperplane arrangement, much topological information is contained in its braid monodromy with respect to a projection transversal to the hyperplanes. In particular the fundamental group is determined by Zariski-van Kampen theorem.

We will explain this and extend the results to quotients by a finite group which act in a way preserving the projection. These results can be applied to give presentations of fundamental groups of complements of discriminants, in unfoldings of isolated singularities and elsewhere.

Saiei-Jaeyeong Matsubara-Heo (Kobe)

Title: Intersection numbers of Euler-Laplace integrals

Abstract:We consider a class of integral representations which we call Euler-Laplace integral. It is canonically isomorphic to GKZ D-module when the parameters are non-resonant. In our setting, GKZ system can be irregular. We investigate the possibility of computing cohomology intersection numbers (c.i.n.) of twisted de Rham cohomology groups of Euler-Laplace integrals. Under a condition, we establish a Laurent expansion formula of the c.i.n.. The key is the construction of a combinatorial basis of a rapid decay homology group. When the c.i.n. is computable by other means, our formula gives rise to new functional identities of hypergeometric functions in several variables.

Keiji Matsumoto (Hokkaido)

1. Title: Monodromy representations for several hypergeometric systems by virtue of intersection forms.

Abstract: There are several hypergeometric systems whose solutions admit Euler type integrals.  
It is classically known that we can obtain a circuit transformation along a loop for each

of such systems by deforming areas of integration and following changes of branches

of an integrand on them along this loop. However, it is difficult to trace these for higher

dimensional cases. In this talk, I give a way to resolve this difficulty. I utilize twisted

homology groups and express circuit transformations as reflections with respect to the

intersection form between them. I illustrate this method by Lauricella's hypergeometric

system F\_D of rank m+1 in m-variables. I introduce some results for monodromy

representations of Lauricella's hypergeometric systems F\_A and F\_C in m-variables of

rank 2^m, and the Aomoto-Gelfand hypergeometric system in (km)-variables of rank

(k+m)!/(k!m!).

References  
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of twisted (co)homology groups. Kyushu J. Math. 67 (2013), no. 2, 367–387.  
[M2] Matsumoto Keiji, Monodromy representations of hypergeometric systems with respect to fundamental series solutions. Tohoku Math. J. (2) 69 (2017), no. 4, 547–570.  
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Monodromy of the hypergeometric differential equation of type (3,6). I.  
Duke Math. J. 71 (1993), no. 2, 403–426.

2) Title: Monodromy representations for several hypergeometric systems by the rigidity.

Abstract: In this talk, we consider the monodromy representation of the generalized hypergeometric differential equation and that of Lauricella's F\_C system of hypergeometric differential equations. We use fundamental systems of solutions expressed by hypergeometric series. We express the circuit matrices along generators of the fundamental group of the complement of the singular locus with respect to each fundamental system of solutions. The aim of this talk is the presentation of a simple way to obtain circuit matrices. We introduce a hypergeometric system in 2 variables of rank 9 found by these considerations.

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Atsuhira Nagano (Kanazawa)

Title: Toric K3 hypersurfaces, hypergeometric systems and their applications to number theory

In this talk, we will consider K3 surfaces given by hyperplanes in toric 3-folds.  
We will study period mappings of such K3 surfaces via GKZ hypergeometric systems.  
Moreover, our story can be considered as a natural extension of the classical arithmetic application of periods of elliptic curves and Gauss hypergeometric equations to imaginary quadratic fields. The speaker will give arithmetic applications of our K3 surfaces.

Noriyuki Otsubo (Chiba)

Title: Hypergeometric motives and p-adic periods

Abstract: In this talk, we discuss possible definitions of motives associated to generalized hypergeometric functions \_{d+1}F\_d, and study their various realizations. Using p-adic Hodge theory, we define p-adic {}\_{d+1}F\_d functions which take values in Fontaine’s ring B\_{dR}, and discuss their relation with hypergeometric functions over finite fields and Dwork’s p-adic hypergeometric functions.

Yota Shamoto (Kavli IPMU)

1. Title: An analogue of Dubrovin’s conjecture.
2. Title: Stokes structures on some differential-difference modules.

Abstract:In this talk, we consider the two parameter families of de Rham cohomology groups associated with pairs of meromorphic functions on Riemann surfaces. The families are endowed with the structures of differential-difference modules. We give a class of solutions to the modules using variants of rapid decay homology groups.The asymptotic behavior can be described in terms of a Stokes filtered constructible sheaves, which we will call q-Stokes structures.

Susumu Tanabé (Galatasaray )

Title: Monodromy of GKZ Hypergeometric functions in terms of Mellin-Barnes integrals

We study the monodromy of Gel’fand-Kapranov-Zelevinski hypergeometric functions that are considered as periods of a Calabi-Yau hypersurface defined by a reflexive polytope. In other words we assume that this polytope gives rise to a reflexive Gorenstein cone. Our principal concern will be the monodromy behaviour of GKZ hypergometric functions as they are analytically continued along loops avoiding discriminantal loci. We shall use the language of amoeba to describe the analytic continuation process. We study the monodromy of Gel’fand-Kapranov-Zelevinski hypergeometric functions that are considered as periods of a Calabi-Yau hypersurface defined by a reflexive polytope. In other words we assume that this polytope gives rise to a reflexive Gorenstein cone. Our principal concern will be the monodromy behaviour of GKZ hypergometric functions as they are analytically continued along loops avoiding discriminantal loci. We shall use the language of amoeba to describe the analytic continuation process. We recall a monodromy formula by Horja that allows interpretation as a variation of Picard-Lefschetz formula in our special setting. We show that in certain cases our main theorem gives supporting evidence for the homological mirror symmetry conjecture by Kontsevich.

Muhammed Uludağ (Galatasaray)

Title: Hypergeometric Galois Actions.

Carlo Verschoor (Utrecht)

Title: A Bailey type factorization of Horn’s H4 hypergeometric function

Abstract: A well-known identity by Bailey states that Appell’s F4 function can be written as the product of two Gauss hypergeometric functions under a suitable specialization of its parameters. Other identities of this type are known for Appell’s F2 and F3, which are closely related to Bailey’s identity. The aim of this talk is to show that the same can be done for Horn’s H4 function.

Yuto Yamamoto (IBS-CGP, Pohang)

Title: Periods of tropical Calabi--Yau hypersurfaces

Abstract: We consider the residual B-model variation of Hodge structure of Iritani defined by a family of toric Calabi--Yau hypersurfaces over a punctured disk D \setminus \{ 0\}. It is naturally extended to a logarithmic variation of polarized Hodge structure (LVPH) of Kato--Usui on D. We also associate an integral affine sphere with singularities B by tropicalization. By using the radiance obstruction of B, we construct a certain polarized logarithmic Hodge structure on the standard log point, which we call the tropical period of B in the talk. We show that the restriction of the LVPH to 0 \in D coincides with the tropical period of B. In order to do it, we compute the radiance obstruction of B. It is expressed as a cohomology Kähler class of its mirror Calabi--Yau hypersurface.

Yadollah Zare (Galatasaray)

Title: Intersection matrices of singularities associated with a pull-back polynomial mapping

Abstract:The method of calculation of the intersection matrix of a singularity of a function of two variables described in this talk is due to S. M. Gusein-Zade and N. A’Campo. This method is based on the product and joining the topology of lower dimension spaces. Their theory can be used to calculate the intersection and Dynkin diagram of pull-back polynomials. Finally, by applying the Picard-Lefschetz formula the action of monodromy group on different kinds of vanishing cycles is investigated.