# Polynomials in two complex variables with two critical values

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From the Belyi polynomial of a planar tree, we can construct a polynomial mapping from  $\mathbf{C}^2$  to  $\mathbf{C}$  with two critical values. We will discuss the monodromy of this family of Riemann surfaces.

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### The homeomorphisms of the space of geodesic laminations and the mapping class group of a surface

#### Charalampos Charitos

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In this work  $S = S_{g,p}$  is an orientable surface of finite type, of genus  $g \geq 0$ with  $p \geq 0$  punctures. We assume furthermore that S is not a sphere with at most four punctures or a torus with at most two punctures or a torus of genus two without punctures. Fixing a hyperbolic metric d on S we consider the set  $\mathcal{GL}(S)$  of geodesic laminations on S with compact support. Consider on  $\mathcal{GL}(S)$  the Hausdorff metric  $d_H$  and denote by  $\mathcal{T}_H$  the Hausdorff topology induced by  $d_H$ . We define also on  $\mathcal{GL}(S)$  another weaker topology  $\mathcal{T}$ , referred as Thurston topology. Any homeomorphism  $h: S \to S$  induces by push-forward a map  $h_*: \mathcal{GL}(S) \to \mathcal{GL}(S)$  which is a homeomorphism with respect to both topologies  $\mathcal{T}_H$  and  $\mathcal{T}$ . The goal of this paper is to prove the following theorem:

**Theorem 1.** Assume that  $f : \mathcal{GL}(S) \to \mathcal{GL}(S)$  is a homeomorphism either with respect to the topology  $\mathcal{T}_H$  or with respect to the topology  $\mathcal{T}$ . Then there is a homeomorphism  $h : S \to S$  such that  $h_* = f$ .

In particular, the theorem above implies that for every isometry f of the metric space  $(\mathcal{GL}(S), d_H)$  there is a homeomorphism  $h: S \to S$  such that  $h_* = f$ .

The result is in the spirit of several rigidity results that were obtained by various authors in the context of mapping class group actions on different spaces.

(joint with I. Papadoperakis and A. Papadopoulos)

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### Metabelian invariants of trigonal curves

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We support the following speculation concerning the fundamental group of a trigonal curve on a Hirzebruch surface:

- 1. There are certain strict bounds on the complexity of the fundamental group of a trigonal curve.
- 2. Any trigonal curve C whose group  $\pi_1$  admits a prescribed quotient G is essentially induced from a certain *universal curve* with this property.
- 3. As a consequence, the existence of a quotient  $\pi_1 \twoheadrightarrow G$  as above may imply certain additional geometric properties of C.
- 4. In particular, the existence of a quotient  $\pi_1 \twoheadrightarrow G$  may imply the existence of a larger quotient  $\pi_1 \twoheadrightarrow \tilde{G} \twoheadrightarrow G$ .

All parts of this speculation are due to the tight relation between trigonal curves, genus zero subgroups of the modular group, and Grothendieck's *dessins d'enfants*.

As an example, we consider metabelian invariants (dihedral quotients and Alexander modules) of the fundamental group. We show that these invariants can take but finitely many values. To illustrate the other parts of the speculation, we prove an analogue of Oka's conjecture on the Alexander polynomial of a plane sextic and discuss the relation between dihedral quotients, Z-splitting sections (in the sense of Shimada), and the Mordell–Weil group of the covering elliptic surface.

We will also discuss the extent to which the fundamental group is controlled by congruence subgroups. (An affirmative answer to this question would imply that only finitely many groups may appear.)

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## Arithmetic and geometry of p-adic triangle groups

### Amir Džambić

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p-adic triangle groups are p-adic analogs of classical (hyperbolic) triangle groups. They have been introduced by Y.Andre and intensively studied by Andre and F.Kato. In my talk I will briefly discuss the basic properties p-adic triangle groups and present an application of them to the study of automorphisms of Mumford curves which is a result of a joint work with Gareth Jones.

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### Comparison of Teichmüller geodesics and Weil-Petersson geodesics

### Ali Gökturk

### $Ko \varsigma \ \ddot{U}niversites i$

Let S be a surface with genus g and n punctures and let x(S) = 3g + n denote the complexity of the surface S. In this talk we prove that in the Teichmüller space T(S), Teichmüller geodesics and Weil-Petersson geodesics with the same pair of end points are fellow-travelers with respect to the Weil-Petersson metric if and only if x(S) < 6. More precisely, we show that if x(S) < 6 then there is a constant N such that for any  $X, Y \in T(S)$ , the Teichmüller geodesic connecting X to Y lies in an N-neighborhood (in the Weil-Petersson metric) of the Weil-Petersson geodesic connecting X to Y. On the other hand we show the opposite of the above statement for surfaces with x(S) > 5.

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### On the uniformization of complex surfaces defined over a number field

### Gabino González-Diez

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A celebrated theorem of Belyi states that a complex curve (= compact Riemann surface) C can be defined by an equation F(x, y) = 0 with coefficients in a number field if and only if  $C \setminus \{\text{finite set}\} \simeq \mathbb{H}/\Gamma$ , for some finite index subgroup  $\Gamma$  of  $PSL(2,\mathbb{Z})$ .

I will speak about a parallel result for complex surfaces in which the role of the upper half plane  $\mathbb{H}$  is played by certain (Bergman) 2-dimensional domains  $\mathbb{B}$  naturally embedded in some Teichmüller space  $T_{g,r}$ , and the uniformising groups are special subgroups of the corresponding mapping class group  $Mod_{g,r}$ . Moreover, results of Shabat seem to indicate that, contrary to the 1-dimensional case, the universal cover  $\mathbb{B}$  already carries information about the arithmeticity of the complex surface in question. The proof uses deep results of Arakelov, Bers, Griffiths and Imayoshi.

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# Geometry and analysis of moduli spaces of Riemann surfaces

### Lizhen Ji

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There has been a lot of work on similarities between symmetric spaces, arithmetic groups and locally symmetric spaces on one hand, and Teichmüller spaces, mapping class groups and moduli spaces of Riemann surfaces on the other hand. In this talk, I will discuss some results in this spirit from the point of views of geometry and analysis.

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### Low-dimensional linear representations of mapping class groups

#### Mustafa Korkmaz

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John Franks and Michael Handel recently proved that, for  $g \ge 3$  and  $n \le 2g-4$ , every homomorphism from the mapping class group of an orientable surface of genus g to  $\operatorname{GL}(n, \mathbb{C})$  is trivial. I will explain the followings:

- 1. This result can be extended to  $n \leq 2g 1$ , also covering the case g = 2.
- 2. Any nontrivial homomorphism from the mapping class group of a surface of genus  $g \geq 3$  to  $\operatorname{GL}(2g, \mathbb{C})$  is conjugate to the standard symplectic representation.
- 3. The mapping class group has no faithful linear representation in dimensions less than or equal to 3g 3.
- 4. A few applications of results in 1 and 2.

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## Efficient computation of the possible topological types of stable, n-pointed curves of genus g

### Stefano Maggiolo

#### SISSA

The moduli space of pointed stable curves  $\overline{M}_{g,n}$  admits a stratification whose strata correspond to the different topological types of stable, *n*-pointed curves of arithmetic genus *g*. One can associate to each such topological type a combinatorial object called a "stable graph". I will present a fast algorithm to generate all possible stable graphs of genus *g* with *n* unordered marked points. An important application of this correspondence is to compute products of tautological classes. (Joint work with Nicola Pagani.)

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## The Galois action on Origami curves and a special class of Origamis

### Florian Nisbach

#### Karlsruher Institut für Technologie

By the term "Origami", or "square tiled surface", we denote a finite covering of an elliptic curve, ramified over at most one point. As Origamis admit a translation structure, a Teichmüller curve in the corresponding moduli space of curves is associated to them. In an article of 2005, Martin Möller gave a construction which associates to a Belyi morphism (or dessin d'enfant) an Origami, in order to show that the action of the absolute Galois group is faithful on the set of Teichmüller curves. We first prove a general inequality exhibiting some connections between arithmetic and geometric properties of Origamis. Then, we study Möller's construction in detail in terms of calculating the genus, the Veech group, the cylinder decomposition etc. of these Origamis, given the combinatorial data of the underlying dessin. This will lead to some series of examples, such as Galois orbits of Origami curves, and an infinite series of noncharacteristic Origamis with Veech group SL(2,Z).

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## Enumerating dessin d'enfants and the moduli space of Riemann surfaces

### Paul Norbury

### University of Melbourne

Dessin d'enfants describe Riemann surfaces defined over the algebraic numbers. The number of dessin d'enfants of a fixed topological type gives the dimension of a representation of the absolute Galois group of the rational numbers. We will show how the enumeration of dessin d'enfants leads to deep information about the moduli space of genus g Riemann surfaces with n labeled points. We will also describe an extension of the count to stable Riemann surfaces.

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## Finsler geometry and Teichmller spaces

Athanase Papadopoulos

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I will give some classical and some new examples of Finsler structures. Some of these structures are related to Teichmüller theory.

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## Gromov-Witten invariants in positive and mixed characteristic

### Flavia Poma

#### SISSA

Gromov-Witten invariants are deformation invariants associated to smooth complex projective varieties, using Kontsevich moduli spaces of stable maps. In a paper by Abramovich and Oort, the authors construct the moduli space of stable maps for projective varieties over an arbitrary base. I will describe how to define Gromov-Witten invariants for smooth projective varieties over any field and, more generally, over a Dedekind domain, focusing on the construction of a virtual fundamental class. I will show that they satisfy the fundamental axioms and some more properties (e.g. WDVV equation, reconstruction theorem). If there is time, I will present a result on the comparison of invariants in different characteristics for smooth projective varieties defined in mixed characteristic.

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## What are correlation functions in higher-genus conformal field theory?

### David Radnell

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A fundamental property of CFT (and vertex operator algebras) is that the correlation (or n-point) functions are "meromorphic functions". Moreover, these functions are required to have a certain factorization property associated to the sewing of Riemann surfaces. The aim of this talk is to explain exactly what type of functions these are in genus-zero, outline the difficulties faced for higher-genus Riemann surfaces and report on some recent progress.

The basic geometric objects in CFT (in the sense of G. Segal) are Riemann surfaces with parametrized boundary components and the associated infinitedimensional rigged moduli space. Alternatively it is possible to use punctured Riemann surface with specified local coordinates at each puncture. Correlation functions are functions on the rigged moduli space. The "meromorphicity" condition is that the function has poles when two punctures coincide. To make sense of this requires a special coordinate system on the rigged Teichmüller space for which the puncture locations form part of the coordinate system. Key tools are Schiffer variation and the lambda-lemma. Solving these foundational problems are necessary steps in the ongoing program to construct CFT from vertex operator algebras.

Joint work with Yi-Zi Huang and Eric Schippers

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### Hurwitz equivalence classes of two Dehn twists and real Lefschetz fibrations

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The topology of Lefschetz fibrations over D2 can be described by the so called monodromy factorization of the monodromy along the boundary of the disk, considered up to a certain sequences of moves, called Hurwitz moves and, possibly, global conjugation. We study factorizations of the monodromy into a product of two Dehn twists in the mapping class group of a torus. First, we give an explicit description of the elements written as a product of two Dehn twists. Then, we give a classification of factorizations up to Hurwitz moves with/without a global conjugation. As an important consequence, we prove that any maximal real elliptic Lefschetz fibration over the sphere is algebraic.

Joint work with A. Degtyarev.

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### Origamis in Teichmüller space

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Teichmüller disks are isometrically (with respect to the Poincar and to the Teichmüller metric) and holomorphically embedded disks in Teichmüller space. A special interest lies on those, whose image in moduli space is an algebraic complex curve, in this case called Teichmüller curve. Teichmüller disks are obtained by a handy construction using translation surfaces. Especially nice examples also called origamis arise from gluing finitely many squares along their edges. Using decorated Teichmüller space, we show how the corresponding Teichmüller disks also naturally live in Culler-Vogtmann outer space  $CV_n$ , the classification space of marked metric graphs of genus n, and give applications of this.

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### Dynnikov Coordinates and Pseudo-Anosov braids

Saadet Öyku Yurttaş

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Isotopy classes of orientation preserving homeomorphisms on punctured disks can be described by braids. The aim of this talk is to present a new method for computing the dilatation of pseudo-Anosov braids making use of Dynnikovs coordinates on the boundary of Teichmller space which is computationally much more efficient than the usual Thurston train track aproach. If time permits, I will talk about some new results on the relation between the spectra of Dynnikov matrices with the spectra of train track transition matrices of a given pseudo-Anosov braid. To be more specific, I will show that these matrices have the same set of eigenvalues up to roots of unity and zeros under some particular conditions.

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