COHEN-MACAULAY PROPERTY FOR SOME NON PRIME BINOMIAL IDEALS RELATED TO COLLECTION OF CELLS.

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We call *cell* a unitary square of $\mathbb{Z}^2$. Considering a finite collection of cells $\mathcal{P}$, we denote by $V(\mathcal{P})$ the set of all vertices of the cells belonging to $\mathcal{P}$. For $a, b \in \mathbb{Z}^2$, with $a < b$ with respect to natural partial order, the set $[a, b] = \{x \in \mathbb{Z}^2 \mid a \leq x \leq b\}$ is called an *inner interval of* $\mathcal{P}$ if every cell contained in it is also contained in $\mathcal{P}$. In [2], A. A. Qureshi showed how to associate a coordinate ring to a collection of cells $\mathcal{P}$. In particular, set $S_\mathcal{P} = K[x_v | v \in V(\mathcal{P})]$, where K is a field, and if $[a, b]$ is an inner interval of $\mathcal{P}$, with a,b and c,d respectively diagonal and anti-diagonal corners, the binomial $x_a x_b - x_c x_d \in S_\mathcal{P}$ is called an *inner 2-minor* of $\mathcal{P}$. Define $I_\mathcal{P}$ the ideal generated by all inner 2-minors of $\mathcal{P}$ and $K[\mathcal{P}] = S_\mathcal{P}/I_\mathcal{P}$, that is, the *coordinate ring* of $\mathcal{P}$. One of the most interesting problem in this context is the study of the Cohen-Macaulay property for the coordinate ring $K[\mathcal{P}]$. In the existing literature, two famous theorems of Sturmfels and Hochster are used to deal the Cohen-Macaulayness in the case $K[\mathcal{P}]$ is a domain. For instance, it is known that every *simple* collection of cells has a Cohen-Macaulay coordinate ring (roughly speaking, a collection of cells is simple if it has not "holes"). No class of collections of cells, whose coordinate rings are Cohen-Macaulay but not domains, has been shown till now.

In our work, we provide some classes of collections of cells whose coordinate rings are Cohen-Macaulay but not domains. In our combinatorial approach, we take advantage of a result in [1], stating that: if I is a homogeneous ideal of a polynomial ring S such that $\text{in}(I)$ is radical with respect to some term order, then $\text{depth}(S/I) = \text{depth}(S/\text{in}(I))$. Furthermore, for the class of *closed path polyominoes*, this approach allows us to give also a complete combinatorial classification of the following algebraic properties and invariants: Gorensteiness, Regularity and Poincarè-Hilbert series.

References

[1] A. Conca, M. Varbaro, *Square-free gröbner degenerations*, Inventiones mathematicae, **221**(3):713–730, 2020.
[2] A.A. Qureshi, Ideals generated by 2-minors, collections of cells and stack polyominoes, J. Algebra 357, 279–303, 2012.

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