# ON THE INTERSECTION DEGREE IN SQUAREFREE VERONESE SUBRINGS

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Let $S \subset \mathbb{N}^n$ be a homogeneous semigroup and let $A = K[S]$ be the $K$-subalgebra of a polynomial ring on a field $K$, generated by the set of monomials which correspond to the generators of degree 1 in $S$. Algebraic and combinatorial properties of $K[S]$ have been studied by many authors (for instance see [1], [2]). An interesting property is strongly Koszulness, introduced and studied in [5]. The homogeneous semigroup $S$ is said to be strongly Koszul if the $K$-algebra $K[S]$ is a strongly Koszul algebra ([5]) and the generators have degree 1. In particular, each $r-$th Veronese subring of $K[x_1, \ldots, x_n]$ is a strongly Koszul algebra. In the squarefree case, the results are partial and obtained for small values of $r$ and $n$ (for $r = 2$ and $n < 5$). In this direction it is very useful to know the so called intersection degree $b(K[S])$ of the semigroup ring $K[S]$, that is the maximum degree of the generators of the colon ideals $(u_i) \cap (u_j), i \neq j$, of $K[S] = K[u_1, u_2, \ldots, u_s]$, where all generators are of degree 1. In [4] we prove that the intersection degree of the third squarefree Veronese subring is 4 for $n \geq 8$, 3 for $n = 5, 6, 7$ and 2 for $n = 4$. So the third squarefree Veronese subring is strongly Koszul for $n = 4$. We develop a procedure using Macaulay2 [6] to find numerical examples for larger $r$ and $n$, in particular we verify the results found in [4] confirming that the squarefree Veronese subring $A^{(n-1,n)}$ is strongly Koszul for any $n$.

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