Toric Codes on Subgroups in a Toric Variety

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After we introduce basics of toric geometry and linear codes, we talk about evaluation codes defined on toric varieties. Evaluating all rational functions from a vector space at a finite subset of $\mathbb{F}_q$-rational points of an algebraic variety gives a linear map whose image is called an evaluation code. Focusing on evaluation codes defined on toric varieties is motivated by the fact that they include champion codes which have the best known parameters. In fact, classical Reed-Solomon codes which have the best possible parameters are just examples in this family viewed as an evaluation code on the projective line.

We show that the number of rational points of a subgroup inside a toric variety over a finite field defined by a homogeneous lattice ideal can be computed via Smith normal form of the matrix whose columns constitute a basis of the lattice. We also prove a Nullstellensatz type theorem over a finite field establishing a one to one correspondence between subgroups of the dense split torus and certain homogeneous lattice ideals. As application, we compute the main parameters of generalized toric codes on subgroups of the torus of Hirzebruch surfaces, generalizing the existing literature. This work is supported by TÜBİTAK Project No:119F177