POLYOCOLLECTION IDEALS AND A COMBINATORIAL DESCRIPTION OF THE PRIMARY DECOMPOSITION

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In 2012 a new topic in Combinatorial Commutative Algebra has emerged by a work of Ayesha Asloob Qureshi. In [2] she establishes a connection between collections of cells and Commutative Algebra, assigning to every collection $\mathcal{P}$ of cells the ideal of the inner 2-minors of $\mathcal{P}$ in a suitable polynomial ring $S_\mathcal{P}$ (see [2]). This ideal $I_\mathcal{P}$ is called the *inner 2-minor* ideal of $\mathcal{P}$ and $K[\mathcal{P}] = S_\mathcal{P}/I_\mathcal{P}$ is said the *coordinate ring* of $\mathcal{P}$. In particular, if $\mathcal{P}$ is a polyomino, that is a collection of cells where the squares are joined edge by edge, then $I_\mathcal{P}$ is called the *polyomino ideal* of $\mathcal{P}$.

The aim of the research is to study the main algebraic properties of $K[\mathcal{P}]$ depending on the shape of $\mathcal{P}$. This has been giving many exciting challenges and one of the most interesting is to provide a combinatorial description of the primary decomposition of $I_\mathcal{P}$.

In [1] we show that for studying the primary decomposition of the polyomino ideals, we should consider a larger class of binomial ideals. This class is related to a new combinatorial object, called *polyocollection*, which generalizes the concept of collection of cells and polyomino. We introduce a binomial ideal attached to a polyocollection, generalizing the ideal associated to a collection of cells in [1]. We provide a characterization of the primality of such a binomial ideal in terms of the lattice ideal attached to the polyocollection and we give a primary decomposition of the radical of that ideal using the so-called admissible sets and the lattice ideals of some suitable polyocollections. Finally, we give a detailed description of the minimal primary decomposition of a particular class of polyominoes, namely closed path polyominoes. We show that the polyomino ideal is the intersection of only two minimal prime ideals and both minimal prime ideals have a very nice combinatorial interpretation in terms of the so-called zig-zag walks and of the vertices in a so-called necklace.

Joint work with Carmelo Cisto and Dharm Veer.

References

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