On curves approaching asymptotic critical value set of a polynomial map

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(with Abuzer GÜNDÜZ)

The bifurcation locus of a polynomial map $f\colon \mathbb{C}^n \to \mathbb{C}$ is the smallest subset $\mathcal{B}(f) \subset \mathbb{C}$ such that f is a locally trivial fibration over $\mathbb{C} \setminus \mathcal{B}(f)$. It is known that $\mathcal{B}(f)$ is the union of the set of critical values $f(\mathrm{Sing} f)$ and the set of bifurcation values at infinity $\mathcal{B}_\infty(f)$ which may be non-empty and disjoint from $f(\mathrm{Sing} f)$ even in very simple examples. Finding the bifurcation locus in the cases $n > 2$ is a difficult task and it still remains to be an unreachable ideal. Nevertheless, one can obtain approximations by supersets of $\mathcal{B}(f)$ by exploiting asymptotical regularity conditions at infinity i.e. by an estimation of the asymptotic critical value set.

In this talk, we show that the asymptotic critical value set of a polynomial map contains the critical values of a polynomial associated to so called "bad face" of the Newton polyhedron. For this purpose, we present an effective method to construct rational curves that make the polynomial to approach asymptotic critical values. In the case when the polynomial map is Newton non-degenerate at infinity, we give also a superset of the asymptotic critical value set including the bifurcation locus. Our main technical tool is the toric geometry that has been introduced into the study of this question by A.Némethi and A.Zaharia.