# Computations With and About the Resultant System

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The resultant is one of the most fundamental objects in computational algebraic geometry. There are plenty of definitions for different cases and even more construction methods. In "Modern Algebra", given a system of polynomials $S$ (without very restrictive conditions on the number of variables or equations) van der Waerden constructs a system of polynomials $R$ with the property that $R$ is feasible if and only if S is feasible. Since this mimics the most intuitive definition of resultant, van der Waerden calls this system "the resultant system". Following the standard construction where the resultant system of a polynomial system of $m$ polynomials $f_i$ is the set of coefficients of $Res\left(\sum_{i=0}^{m} u_i f_i, \sum_{i=0}^{m} v_i f_i, \right)$, when viewed as a polynomial in the $u_i$ and $v_i$ variables, the number of elements in the resultant system is enormous. Nevertheless, we observe that there is a large number of repetitions due to symmetry. Moreover, some of the coefficients are combinations of others and thus not needed in the resultant system. After an analysis of such redundancies and structure, we argue that the resultant system is a useful tool in real algebraic geometry, both in theory and in practice. In computing elimination ideals, we show that for some cases computing the resultant system first improves the performance.