Université Galatasaray, Département de Mathématiques
2012 - Second Semestre – Math 468 - Théorie de Corps
Examen Final, 05 juin 2013 – Ayberk Zeytin
120 mins. Nom & Prénom: ____________________

Question:	1	2	3	4	5	Total
Points:	32	20	48	10	24	134
Score:						

Question 1 (32 points)

True or false:

(a) (8 points) ______ $\mathbf{Q}(\pi)$ is a splitting field over $\mathbf{Q}(\pi^2)$.

(b) (8 points) ______ Every finite extension of every field k is separable over k.

(c) (8 points) ______ For any *subset* $\mathsf{A} \subset \mathrm{Aut}(\mathsf{K}/\mathsf{k})$, the set of elements in K which are fixed by A is a subfield of K.

(d) (8 points) ______ The fields $\mathbf{Q}(\sqrt{2})$ and $\mathbf{Q}(\sqrt{3})$ are isomorphic.

Question 2 (20 points)

Show that if $[\mathsf{K} : \mathsf{k}] = 2$ then K is a splitting field over k.

Question 3 (48 points)

Let K be the splitting field of the polynomial $f(X) = X^8 - 2$ in $\mathbf{C}$. Let $\alpha = \sqrt[8]{2}$.

(a) (8 points) Show that $K = \mathbf{Q}(\alpha, \zeta_8)$.

(b) (8 points) Show that $\mathbf{Q}(\zeta_8) = \mathbf{Q}(\sqrt{2}, \sqrt{-1})$ deduce that $K = \mathbf{Q}(\alpha, \sqrt{-1})$.

(c) (8 points) Show that $\mathrm{Aut}(\mathsf{K}/\mathbf{Q})$ contains an automorphism, say σ, of order 8.

(d) (8 points) Show that $\mathrm{Aut}(\mathsf{K}/\mathbf{Q})$ contains an automorphism, say τ, of order 2.

(e) (8 points) Find the field fixed by the subgroup generated by σ, i.e. find $\mathrm{Fix}(\langle\sigma\rangle)$.

(f) (8 points) Find the field fixed by the subgroup generated by τ, i.e. find $\mathrm{Fix}(\langle\tau\rangle)$.

Question 4 (10 points)

Find an extension K of $\mathbf{Q}$ with the property that $|\mathrm{Aut}(\mathsf{K}/\mathbf{Q})| = 3$.

Question 5 (24 points)

Let $\mathsf{K} = \mathbf{C}(z, \sqrt{z}, \sqrt[3]{z})$ be the rational function field in z, $\sqrt{z}$ and $\sqrt[3]{z}$.

(a) (8 points) Show that the extension $\mathsf{K}/\mathbf{C}(z)$ is of degree 6.

(b) (8 points) Find the group $\mathrm{Aut}(\mathsf{K}/\mathbf{C}(z))$ and deduce that this extension is Galois.

(c) (8 points) Take an order 2 element, call σ, in $\mathrm{Aut}(\mathsf{K}/\mathbf{C}(z))$ and let $\mathsf{G} = \{1, \sigma\}$. Find $\mathrm{Fix}(\mathsf{G})$.