MATH 501
EXERCISES 7

A. ZEYTİN

Throughout by $\mathbf{C}$ ($\mathbf{P}^1$, resp.) we denote the field of complex numbers (Riemann sphere, resp.) and z is always a point of $\mathbf{P}^1$. By $\varphi\colon S^2 \setminus \{N = (0,0,1)\} \longrightarrow \mathbf{C}$ we denote the stereographic projection and by ψ its inverse. Ω denotes a lattice in $\mathbf{C}$ generated by two $\mathbf{R}$-linearly independent complex numbers ω_1 and ω_2.

(1) Say γ_0, γ_1 and γ_2 are three paths from a to b in $X \subset \mathbf{P}^1$ so that Γ_1 defines a homotopy between γ_0 and γ_1 and Γ_2 defines a homotopy between γ_1 and γ_2. Show that there exists a homotopy Γ_3 between γ_0 and γ_2, and hence deduce that homotopy equivalence is an equivalence relation.

(2) Describe the Riemann surface of the functions $\sqrt{(z-a)^2}$, where $a \in \mathbf{C}$.

(3) Construct the Riemann surface of $\arcsin(z)$.

(4) Find the position and order of the branch points of the following functions:
- $\sqrt{\prod_{i=1}^{5}(z-i)}$
- $\log(\sin(z))$
- $\sqrt{1-z^3}$

Further, in each case, determine a *largest* cut plane so that the function admits a branch.