MATH 532
EXERCISES 4

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Exercises marked with $*$ are your homework.

(1) $*$ Let L_1 be the line $X + Y$ and L_2 be the line $X - Y$.
- ▶ Construct an algebraic curve, denoted by F for which L_1 is a tangent line of multiplicity 3, L_2 is a tangent line of multiplicity 2, and F is irreducible. Hint: Use higher order terms to make F irreducible.
- ▶ Construct an algebraic curve, denoted by F for which L_1 is a tangent line of multiplicity r_1, L_2 is a tangent line of multiplicity r_2, and F is irreducible.
- ▶ Repeat the question with arbitrary but finite number of lines with arbitrary but finite multiplicities.[1]

(2) Let $\mathcal{O}$ be a local ring and $\mathfrak{m}$ be its unique maximal ideal. Show that the following sequence is exact :

$$0 \longrightarrow \mathfrak{m}^n/\mathfrak{m}^{n+1} \longrightarrow \mathcal{O}/\mathfrak{m}^{n+1} \longrightarrow \mathcal{O}/\mathfrak{m}^n \longrightarrow 0.$$

Compute the above sequence for $n = 1, 2, 3$ explicitly for the following curves:
- ▶$*$ $Y - X^2$,
- ▶ $Y - X^3$,
- ▶ $Y^2 - X^3$.

(3) The following questions are assigned from our textbook which is available at

http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf

2.14, 2.15, 2.22, 2.34, 3.8, 3.9, 3.10, 3.11, 3.12, 3.13$*$, 3.14[2], 3.15,

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[1] You'll need to solve Problem 2.34 from our textbook beforehand for this part of the question.
[2] You'll need to solve Problem 1.40 for this one, too.