MATH 532
EXERCISES 5

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Exercises marked with $*$ are your homework.

(1) *Compute the intersection multiplicity of the curves $F = XY^3 + Y + X^3$ and $G = 3Y + X^3$, at the point $(0, 0)$. (In fact, you can compute all the other intersection points explicitly...)

(2) *Compute the intersection multiplicity of the curves $F = Y^2 - X^3 + X^2$ and $G = Y$, at the point $(0, 0)$.

(3) Let $F = aX + bY + c$ and $G = aX + bY + d$ de two lines in $\mathbb{A}^2$; with $c \neq d$, so that F and G are parallel lines; hence they do not intersect. Decide whether F^* and G^* intersect in $\mathbf{P}^2$. (Recall that F^* is our notation for the homogenization of F.)

(4) Show that if I and J are homogenous ideals of $k[X_0, \ldots, X_n]$; then
- ▶ $I + J$
- ▶ IJ
- ▶ $I \cap J$

are homogenous ideals, too.

(5) Determine
- ▶ $V_p(Y^2, X)$, $V_a(Y^2, X)$
- ▶ $V_p(aX - bY)$, $V_a(aX - bY)$, $V_a(aX - b)$
- ▶ $V_p(X - Y, X^2 - YZ)$, $V_a(X - Y, X^2 - Y)$

(6) Is $V_p(XZ^3 + Y^2Z^2 - X^3Z - X^2Y^2)$ irreducible?

(7) Compare the points of
- ▶ *$V_1 = V_a(Y - X^2) \subset \mathbb{A}^2$ and $V_1' = V_p(ZY - X^2) \subset \mathbf{P}^2$
- ▶ $V_1 = V_a(XY - 1) \subset \mathbb{A}^2$ and $V_1' = V_p(XY - Z^2) \subset \mathbf{P}^2$

<u>Hint:</u> Use the maps φ_i introduced in the lecture...

(8) The following questions are assigned from our textbook which is available at

http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf

4.1, 4.2, 4.3, 4.4, 4.5*, 4.6, 4.9*