MATH 532
EXERCISES 6

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Exercises marked with $*$ are your homework.

(1) *How does the projetive curve $y^2 - x^2 + (z-x)^2$ look in each of the charts of $\mathbf{P}^2$.

(2) If X is a projective variety, determined by the homogeneous ideal I, then its coordinate ring, $R = k[x_0, \ldots, x_n]/I$, is a *graded ring*, i.e. it is a sum of vector spaces

$$R = R_0 \oplus R_1 \oplus \ldots$$

so that $R_i R_j \subset R_{i+j}$; where each R_i is comprised the residues of degree i forms in R.

- ▶ Show indeed that R_i is a vector space over k.
- ▶ Show that none of the elements of R_i can be regarded as a function on X. Hint: They are not well-defined.
- ▶ For $I = X^2 + Y^2 - Z^2$; find the basis for R_i for $i = 0, 1, 2$.

(3) A hypersurface in $\mathbf{P}^n$ is defined as the zero set of a single homogeneous polynomial in $k[x_0, \ldots x_n]$. A hypersurface which is the zero set of a homogeneous polynomial of degree 1 (i.e a *linear polynomial*) is called a hyperplane. Show that the following conditions on a hypersurface X are equivalent:

i. $I(X)$ can be generated by linear polynomials
ii. X can be written as intersection of hyperplanes

(4) The following questions are assigned from our textbook which is available at

http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf

4.10, 4.11, 4.12, 4.13, 4.14, 4.15* (do NOT use Bezout's theorem), 4.16, 4.17, 4.19, 4.20, 4.22, 4.23, 4.24, 4.25*, 5.2, 5.3, 5.4*, 5.5, 5.6, 5.7*(try to use the affine version of the theorem and Exercise 4.25), 5.8, 5.14, 5.15, 5.16, 5.18*, 5.19, 5.21