Université Galatasaray, Département de Mathématiques
2016 - Spring Semester – Math 532 - Selected Topics in Algebraic Geometry
Final Exam, 26 May 2017 – Ayberk Zeytin, 120 minutes
Name & Surname: ____________________ **Sign:** __________

Question:	1	2	3	Total
Points:	8	6	4	18
Score:				

Question 1 (8 points)
We fix our ground field to be **C** (or any other algebraically closed field of characteristic 0.)

(a) (2 points) Recall the definition of the cone over an affine variety W.

(b) (2 points) Let W_1 be the parabola $V(y - x^2) \subseteq \mathbb{A}^2$. Draw the cone over W_1 and identify the line corresponding to the *point at infinity* on the projectivization of W_1.

(c) (2 points) Show that the affine varieties $W_1 = V(y - x^2)$ and $W_2 = V(y - x^3)$ are isomorphic.

(d) (2 points) Decide whether their projective closures have isomorphic coordinate rings.

Question 2 (6 points)

Consider the projective curve $\mathsf{F}(\mathsf{X},\mathsf{Y},\mathsf{Z}) = \mathsf{Y}^2\mathsf{Z}^2 + \mathsf{Z}^2\mathsf{X}^2 + \mathsf{X}^2\mathsf{Y}^2 2\mathsf{XYZ}(\mathsf{X}+\mathsf{Y}+\mathsf{Z})$. This curve is called the *tricuspidal quartic.*

(a) (2 points) Find an equation of the tangent line, say L, to the curve at $\mathsf{P} = (1:0:0)$, algebraically.

(b) (2 points) Show that the intersection multiplicity $\mathrm{I}(\mathsf{P}, \mathsf{F}\cap\mathsf{L}) = 3$. Such a point is called a cusp. Using symmetry of the equation, find the remaining two cusps(hence the name tricuspidal!). Name these points Q and R.

(c) (2 points) Find a quadric (i.e. a degree two curve in $\mathbf{P}^2$ - also called a conic because of its dimension) G that passes through the points P, Q and R. Decide whether F and G intersect in another point or not.

Question 3 (4 points)

Let L_1, L_2 be two disjoint lines in $\mathbf{P}^3$ and fix a point $a \in \mathbf{P}^3 \setminus (L_1 \cup L_2)$.

(a) (2 points) Show that there is a *unique* line L that passes through a, which intersects both L_1 and L_2.

(b) (2 points) Is the result true if we replace $\mathbf{P}_3$ by $\mathbb{A}^3$.