Université Galatasaray, Département de Mathématiques
2016 - Spring Semester – Math 532 - Selected Topics in Algebraic Geometry
Mid Term Exam, 06 April 2017 – Ayberk Zeytin, 90 minutes
Name & Surname: ______________________ **Sign:** __________

Question:	1	2	3	Total
Points:	6	4	2	12
Score:				

Question 1 (6 points)

Let k be an algebraically closed field.

(a) (2 points) Show that the circle $\mathsf{V}(\mathsf{X}^2 + \mathsf{Y}^2 - 1)$ is isomorphic (as an affine variety) to the affine curve (in fact the hyperbola) $\mathsf{V}(\mathsf{XY} - 1)$.

(b) (2 points) Show that $\mathsf{V}(\mathsf{XY} - 1)$ is an irreducible algebraic set. Deduce that circle is also irreducible.

(c) (2 points) Show that neither is isomorphic to $\mathbb{A}^1(\mathsf{k})$.

Question 2 (4 points)

Let k be an infinite field (not necessarily algebraically closed).

(a) (2 points) Show that an $f \in k[X_1, \ldots, X_n]$ that is identically zero on $\mathbb{A}^n(k)$ is the zero polynomial (i.e., has all its coefficients zero).

(b) (2 points) Decide whether the claim is true if k is a finite field.

Question 3 (2 points)

Let k be a field (not necessarily algebraically closed). The set of *formal power series* over k, written $k[[X]]$ is defined to be :

$$k[[X]] := \left\{ \sum_{i=0}^{\infty} a_i X^i \mid a_i \in k \right\}.$$

This set becomes a ring under usual addition and multiplication:

$$\begin{aligned} \sum_{i=0}^{\infty} a_i X^i + \sum_{i=0}^{\infty} b_i X^i &= \sum_{i=0}^{\infty} (a_i + b_i) X^i \\ \left(\sum_{i=0}^{\infty} a_i X^i \right) \left(\sum_{i=0}^{\infty} b_i X^i \right) &= \sum_{i=0}^{\infty} \left(\sum_{m=0}^{i} a_m b_{i-m} \right) X^i \end{aligned}$$

Note that $k[[X]]$ contains $k[X]$ as a subring. Show that $k[[X]]$ is a discrete valuation ring with local parameter X.