

**MATH 532**  
**EXERCISES 4**

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Unless otherwise stated  $K$  and  $k$  are fields and  $K/k$  is a field extension.

- (1) Let  $K/k$  be a field extension of prime degree. What can you say about its automorphism group?
- (2) Determine all primitive roots of unity in the field  $\mathbf{F}_3[X]/(X^2 + 1)$ .
- (3) For the following field extensions ( $K/k$ ) describe the subgroup lattice of the Galois group and make the Galois correspondence explicit :
  - ▶  $K = \mathbf{Q}(\sqrt[4]{2}, \sqrt{-1}), k = \mathbf{Q}$
  - ▶  $K = \mathbf{Q}(\zeta_7), k = \mathbf{Q}$
  - ▶  $K = \mathbf{Q}(\zeta_8), k = \mathbf{Q}$
  - ▶  $K = \mathbf{Q}(\zeta_{12}), k = \mathbf{Q}$
  - ▶  $K/\mathbf{Q}$ ; where  $K$  is the splitting field of the polynomial  $X^8 - 2$
  - ▶  $K = \mathbf{F}_{p^{15}}, k = \mathbf{F}_p$