Université Galatasaray, Département de Mathématiques
2018 - Spring Semester – Math 532 - Selected Topics in Algebraic Geometry
Mid Term Exam, 18 April 2018 – Ayberk Zeytin, 120 minutes
Name & Surname: ____________________ **Sign:** __________

Question:	1	2	3	Total
Points:	5	6	6	17
Score:				

Question 1 (5 points)
Decide whether the following statements are true or false. Give a very short (one or two sentence) explanation (reason if true, counter-example if false) for each:

(a) (True / False) Any finite extension is algebraic.

(b) (True / False) Any algebraic extension is finite.

(c) (True / False) For any field k, the extension $\overline{k}/k$ is infinite; where $\overline{k}$ denotes the algebraic closure of k.

(d) (True / False) $\mathbf{C}$ is an algebraic closure of $\mathbf{Q}$.

(e) (True / False) Given any algebraic extension $K/\mathbf{Q}$ there is an injective ring homomorphism $K \to \mathbf{C}$.

(f) (True / False) $\mathbf{F}_{p^2}$ can be embedded into $\mathbf{F}_{p^3}$.

(g) (True / False) An algebraic closure of $\mathbf{F}_p$ can contain many different subfields of order p^n for some $n \in \mathbf{N}$.

(h) (True / False) For every $n \in \mathbf{N}$ there is an irreducible polynomial $p(X) \in \mathbf{F}_p[X]$ of degree n.

(i) (True / False) Let $K = \overline{\mathbf{F}_p(t)}$ and let $\mathbf{F}_p(\alpha)$ be a stem field for $X^p - t \in \mathbf{F}_p(t)[X]$. There are p-many $\mathbf{F}_p(t)$ homomorphisms $\mathbf{F}_p(\alpha) \to K$.

Question 2 (6 points)

Let $k = \mathbf{F}_p$ and let $f(X) \in k[X]$.

(a) (2 points) Show that if $f'(X) = 0$ then $f(X)$ is reducible.

(b) (2 points) Deduce that if $f(X) \in k[X]$ is irreducible then $f(X)$ is separable.

(c) (2 points) Prove that finite fields are perfect. That is, let K be a finite extension of k. Prove that if $f(X) \in K[X]$ is irreducible then f is separable. (`Hint:` Recall that perfect fields are those in which every element is a p^{th} power.)

Question 3 (6 points)

Work out the explicit Galois correspondence for $\mathbf{Q}(\zeta_9)/\mathbf{Q}$.