MATH 519
EXERCISES 2

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The following numbered are exercises from our textbook : An Invitation to Arithmetic Geometry, *D. Lorenzini*, AMS GSM series vol.9 1997

(1) Use Zorn's lemma to show that every ideal of a commutative ring R with unity is contained in a maximal ideal.

(2) Let R be a local ring with $\mathfrak{m}$ its unique maximal ideal.
- ▶ Show that every element of $R \setminus \mathfrak{m}$ is a unit.
- ▶ Show that for any $x \in R$ either x is invertible or $1 - x$ is invertible, or both.

(3) Let R be an integral domain in which all prime ideals are principal. Show that the ring is a principal ideal domain.

(4) Can you find a non-constant unit in C_f where $f = x^2 + y^2 - 1$.

(5) Let $f \in k[x, y]$; where k is an algebraically closed field. Decide whether the element $x - a$ can ever be an invertible element of $C_f := k[x, y]/(f)$.

(6) Chapter II. 1, 9, 10, 12, 17

Among these 4, 5, II. 9, 10, are your homework to be submitted due 29/11/2019.