MATH 504
EXERCISES 5

A. ZEYTİN

Unless otherwise stated G is a group.

(1) Use 1, 2, 3 to number the opposite faces of a cube in $\mathbf{R}^3$, say the front and back faces with 1, the left and right faces with 2, top and bottom faces with 6. Consider the action of $\mathfrak{S}_3$ on these labels, as rigid motions. Describe the corresponding permutation representation. What is the kernel of this action? Is it transitive? Is it faithful?

(2) Consider the action of $\mathfrak{S}_6$ on the faces of a dice, numbered from 1 to 6, by acting on these numbers. Can one describe this action by rigid motions? Explain.

(3) Describe the permutation representation of the action of $\mathfrak{S}_3$ on itself by multiplication from left. What is the kernel of this action? Is it transitive? Is it faithful?

(4) Let $G = \mathbf{Z}/7\mathbf{Z}$. Describe the permutation representation of the action of G on itself by multiplication from left. What is the kernel of this action? Is it transitive? Is it faithful?

(5) Let $G = \mathbf{Z}$. Describe the permutation representation of the action of G on itself by multiplication from left. What is the kernel of this action? Is it transitive? Is it faithful?

(6) We let $e_1 = (1\,0\,0)^t$, $e_2 = (0\,1\,0)^t$, $e_3 = (0\,0\,1)^t$ be the standard basis of $\mathbf{R}^3$.
- ▶ Show that the set $Q_8 = \{\pm 1, \pm e_1, \pm e_2, \pm e_3\}$ endowed with the standard cross product for the basis vectors e_1, e_2 and e_3 extended obviously to include ± 1 is a group. This group is called the *quaternion group*.
- ▶ Show that this group is not abelian.
- ▶ Describe the permutation representation associated to the action of Q_8 on itself by multiplication from left.
- ▶ Compute the orbit of e_1. Is the action transitive?
- ▶ Is the mentioned action faithful?

(7) Let G be a group acting on a set X. Show that G acts faithfully on X if and only if no two distinct elements of G have the same action on each element of X.

(8) Show that A_n, the subgroup of even permutations in $\mathfrak{S}_n$, is a normal subgroup.