MATH 518
EXERCISES 7

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1. Write a double integral and calculate (if possible) areas of the following regions in $\mathbb{H}$:
   - ▶ the region bounded by $\{z \in \mathbb{H} \mid |\mathrm{re}(z)| = 1\}$, and $\{z \in \mathbb{H} \mid |z| = 1\}$.
   - ▶ the region bounded by $\{z \in \mathbb{H} \mid |\mathrm{re}(z)| = s\}$, and $\{z \in \mathbb{H} \mid |z| = s\}$; where $s \in \mathbf{R}_+$. Investigate what happens if $s \to \infty$.
   - ▶ the region bounded by $\{z \in \mathbb{H} \mid |z| = 1\}$, $\{z \in \mathbb{H} \mid |z| = 2\}$, $\{z \in \mathbb{H} \mid \arg(z) = \pi/4\}$ and $\{z \in \mathbb{H} \mid \arg(z) = 3\pi/4\}$.
   - ▶ the region bounded by $\{z \in \mathbb{H} \mid |z| = 1\}$, $\{z \in \mathbb{H} \mid |z| = 2\}$ and $\{z \in \mathbb{H} \mid |\arg(z) - \pi/2| = \theta\}$; where $\theta \in (0, \pi/2)$. Investigate what happens if $\theta \to 0$ and if $\theta \to \pi/2$.
   - ▶ the region bounded by $\{z \in \mathbb{H} \mid |z| = 1\}$, $\{z \in \mathbb{H} \mid |z| = 2\}$, $\{z \in \mathbb{H} \mid |z - 3/2| = 1/2\}$ and $\{z \in \mathbb{H} \mid |z + 3/2| = 1/2\}$.
   - ▶ the region bounded by $\{z \in \mathbb{H} \mid |z| = 1/2\}$, $\{z \in \mathbb{H} \mid |z - 1| = 1/2\}$, $\{z \in \mathbb{H} \mid |z + 1/2| = 1/2\}$ and $\{z \in \mathbb{H} \mid |z| = 3/2\}$.

2. Write a double integral and calculate (if possible) areas of the following regions in $\mathbb{D}$:
   - ▶ the region bounded by $\{z \in \mathbb{D} \mid \mathrm{re}(z) = 0\}$, $\{z \in \mathbb{D} \mid \mathrm{im}(z) = 0\}$ and $\{z \in \mathbb{D} \mid |z - (1 + i)| = 1\}$.
   - ▶ the region bounded by $\{z \in \mathbb{D} \mid \mathrm{re}(z) = 0\}$, $\{z \in \mathbb{D} \mid \mathrm{im}(z) = 0\}$, $\{z \in \mathbb{D} \mid \mathrm{re}(z) = 1/2\}$ and $\{z \in \mathbb{D} \mid \mathrm{im}(z) = 1/2\}$.
   - ▶ the region bounded by $\{z \in \mathbb{D} \mid \mathrm{re}(z) = 0\}$, $\{z \in \mathbb{D} \mid \mathrm{im}(z) = 0\}$, $\{z \in \mathbb{D} \mid \mathrm{re}(z) = s\}$ and $\{z \in \mathbb{D} \mid \mathrm{im}(z) = 1/2\}$; where $s \in (0, 1)$. Investigate what happens when $s \to 0$ and when $s \to 1$.
   - ▶ the region bounded by $\{z \in \mathbb{D} \mid \mathrm{re}(z) = 0\}$, $\{z \in \mathbb{D} \mid \mathrm{im}(z) = 0\}$, $\{z \in \mathbb{D} \mid \mathrm{re}(z) = 1/3\}$ and $\{z \in \mathbb{D} \mid \mathrm{im}(z) = s\}$; where $s \in (0, 1)$. Investigate what happens when $s \to 0$ and when $s \to 1$.