MATH 518
EXERCISES 9

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1. Using classification of elements of $\mathrm{SL}(2, \mathbf{R})$, give a classification of elements of $\mathrm{SU}(1,1)$.
2. Classify the following Möbius transformations :
   - $f(z) = \dfrac{3z-4}{4z+6}$
   - $f(z) = \dfrac{-1}{z}$
   - $f(z) = \dfrac{2z+5}{-z+7}$
   - $f(z) = \dfrac{z-1}{z+1}$
   - $f(z) = \dfrac{-z}{z-1}$
3. In this exercise, we are going to show that if a Fuchsian group is Abelian then it is cyclic. Let $A, B \in \mathrm{SL}(2, \mathbf{R})$.
   - Show that if A and B commute then B maps the fixed point set of A to itself. Moreover, this mapping is injective.
   - Show that if $AB = BA$, then fixed points of A are exactly the same as fixed points of B.
   - Show that if A and B have exactly the same fixed point set, then $AB = BA$
   - Show that if G is an Abelian Fuchsian group, then show that all elements of G are of the same class, i.e. all elements are parabolic, elliptic or hyperbolic.
   - Show that if all elements of and Abelian Fuchsian group G are hyperbolic then G is cyclic.
   - Show that if all elements of and Abelian Fuchsian group G are parabolic then G is cyclic.
   - Show that if all elements of and Abelian Fuchsian group G are elliptic then G is cyclic.
   - Conclude that $\mathrm{SL}(2, \mathbf{R})$ does not have any subgroup which is isomorphic to $\mathbf{Z} \otimes \mathbf{Z}$
4. Let G be a Fuchsian group in $\mathrm{SL}(2, \mathbf{R})$. Show that if there is a sequence, A_n in G which converges to I, that is $\lim_{n\to\infty} A_n = I$, then there is some $N \in \mathbf{N}$ so that $A_n = I$ for all $n > N$.
5. Prove that multiplication and taking inverses are continuous mappings from $\mathrm{SL}(2, \mathbf{R})$ to itself.
6. Show that the set
$$G = \{A = \begin{pmatrix} a_1 + a_2\sqrt{2} & b_1 + b_2\sqrt{2} \\ c_1 + c_2\sqrt{2} & d_1 + d_2\sqrt{2} \end{pmatrix} \in \mathrm{SL}(2, \mathbf{R}) \mid \det(A) = 1\}$$
   is a subgroup of $\mathrm{SL}(2, \mathbf{R})$. Show, however, that G is not a Fuchsian group.