MATH 201
ÉNONCÉS DES EXERCICES 12

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(1) Trouver les points critiques, singulières et frontières de la fonction f et déterminer les points auxquelles f admet les valeurs extremes sur D.

- $f(x,y) = x^2 + 2xy - y^2, \quad D = \{(x,y) \in \mathbf{R}^2 \mid x^2 + y^2 \leq 1\}$
- $f(x,y) = \sqrt{x+2y}, \quad D = \{(x,y) \in \mathbf{R}^2 \mid x^2 + y^2 \leq 9\}$
- $f(x,y) = xy - y^2, \quad D = \{(x,y) \in \mathbf{R}^2 \mid x^2 + y^2 \leq 1\}$
- $f(x,y) = 2xy + x^2, \quad D = \{(x,y) \in \mathbf{R}^2 \mid -1 \leq x \leq 1, x-1 \leq y \leq -x+1\}$
- $f(x,y) = x^2 + 2y - 5xy, \quad D = \{(x,y) \in \mathbf{R}^2 \mid y = 1-x, y = 1+x, y = -1-x, y = -1+x\}$
- $f(x,y,z) = x^2 + y^2 + z^2 - x + y, \quad D = \{(x,y,z) \in \mathbf{R}^3 \mid x^2 + y^2 + z^2 \leq 1\}$
- $f(x,y) = 3x^2 + y^2 - 3xy, \quad D = \{(x,y) \in \mathbf{R}^2 \mid y = 3, y = x^2\}$
- $f(x,y) = x^2 - xy + y^2, \quad D = \{(x,y) \in \mathbf{R}^2 \mid x^2 + y^2 \leq 4\}$
- $f(x,y) = x^2ye^{-(x+y)}, \quad D = \{(x,y) \in \mathbf{R}^2 \mid x \geq 0, y \geq 0, x+y \leq 5\}$
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- $f(x,y,z) = x^4 + y^4 + z^4, \quad D = \{(x,y,z) \in \mathbf{R}^3 \mid x^2 + y^2 + z^2 = 1\}$
- $f(x,y) = xy, \quad D = \{(x,y) \in \mathbf{R}^2 \mid \dfrac{x^2}{16} + y^2 \leq 1\}$
- $f(x,y) = x^3 - x + y^2 - xy, \quad D = \{(x,y) \in \mathbf{R}^2 \mid x \geq 0, y \geq 0, x+y \leq 2\}$
- $f(x,y) = xy - x^3y^2, \quad D = \{(x,y) \in \mathbf{R}^2 \mid x \geq 0, y \geq 0, x+y \leq 2\}$
- $f(x,y) = \dfrac{x-y}{1+x^2+y^2}, \quad D = \{(x,y) \in \mathbf{R}^2 \mid y \geq 0\}$
- $f(x,y,z) = x^2 + y^2 + z^2, \quad D = \{(x,y,z) \in \mathbf{R}^3 \mid x - y = 1, y^2 - z^2 = 1\}$
- $f(x,y) = 4x + 6y - x^2 - y^2, \quad D = \{(x,y) \in \mathbf{R}^2 \mid 0 \leq x \leq 4, 0 \leq y \leq 5\}$

(2) Pour les formes quadratiques suivantes, trouver la matrice associée et déterminer elle est définis positif, négatif ou bien indéfini:

- $Q(x,y) = 8xy - x^2 - 30y^2$
- $Q(x,y,z) = 3x^2 - 2xy + 4xz + 5y^2 + 4z^2 - 2yz$
- $Q(x,y,z) = 5x^2 + 3y^2 + 2z^2 - xy + 8yz$
- $Q(x,y) = x^2 - 3y^2 + 7xy$
- $Q(x,y,z) = 9x^2 + 7y^2 + 3z^2 - 2xy + 4xz - 6yz$
- $Q(x,y,z) = x^2 - 3yz$