

MATH 201
ÉNONCÉS DES EXERCICES 4

A. ZEYTIN

(1) Trouver l'intérieur et l'adhérence des ensembles suivantes dans \mathbf{R}^n :

- ▶ $E = \{(x, y) \in \mathbf{R}^2 \mid 0 < x < 2 \text{ et } 0 \leq y \leq 1\}$
- ▶ $E = \{(x, y) \in \mathbf{R}^2 \mid 0 \leq x^2 + y^2 \leq 1\}$
- ▶ $E = \{x \in (0, 1) \mid x \text{ est rationnelle}\}$
- ▶ $E = \{(x, y) \in \mathbf{R}^2 \mid 0 < x \leq 1\}$
- ▶ $E = \{(x, y) \in \mathbf{R}^2 \mid 0 < x \leq 1, y = 0\}$
- ▶ $E = \{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 \leq 1, x + y + z = 1\}$
- ▶ $E = \mathbf{R}^2 - ([0, 1] \cup (2, 3))$
- ▶ $E = [1, 2) \cup (2, 5) \cup \{10\}$
- ▶ $E = \{(x, y) \in \mathbf{R}^2 \mid x \leq 2 - y\}$
- ▶ $E = \{(x, y) \in \mathbf{R}^2 \mid y \leq |x|\}$
- ▶ $E = \{(x, y, z) \in \mathbf{R}^3 \mid 0 < x^2 + y^2 + z^2 < 4\}$
- ▶ $E = \{(x, y, z) \in \mathbf{R}^3 \mid (x - 1)^2 + y^2 + z^2 < 1\} \cup \{(x, y, z) \in \mathbf{R}^3 \mid (x + 1)^2 + y^2 + z^2 \leq 1\}$
- ▶ $E = \{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 + z^2 \geq 4\} \cap \mathbf{Q}^2$
- ▶ $E = \{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 < 1, |z| < 1\}$

(2) Soit $E \subset \mathbf{R}^n$. Soit $E^c = \mathbf{R}^n - E$. Montrer que

- ▶ $\overline{\overline{E^c}} = ((E^c)^\circ)^\circ$
- ▶ $E^\circ = (\overline{\overline{E^c}})^\circ$
- ▶ $\overline{E^c} = (E^\circ)^c$
- ▶ $\overline{E^c} = (E^c)^\circ$
- ▶ $(E - E^\circ)^\circ = \emptyset$

(3) Trouver les points isolés et d'accumulations des ensembles suivantes:

- ▶ $(-1, 1)$
- ▶ $[-1, 1]$
- ▶ \mathbf{N}
- ▶ \mathbf{R}
- ▶ \mathbf{Q}
- ▶ $(-1, 1) \cup [0, 1]$
- ▶ $(-1, 1) - \{\frac{1}{2}\}$
- ▶ $(0, 1] \cup \{2, 3\}$
- ▶ $\{(x, y, z) \in \mathbf{R}^3 \mid 0 \leq x < 1, 0 < y < 1, 0 < z < 2x, y, z \in \mathbf{Q}\}$
- ▶ $\{(x, y) \in \mathbf{R}^2 \mid x = 0, 0 < y < 1\}$
- ▶ $\{(x, y) \in \mathbf{R}^2 \mid x \in \mathbf{Z}, y \in \mathbf{Z}\}$
- ▶ $\{5\}$