MATH 504
EXERCISES 1

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Unless otherwise stated G is a group.

(1) Let $G = C^0((0,1))$. On G we define the usual composition as well as pointwise addition and multiplication on G as follows :

$$+ \colon G \times G \to G$$
$$(f,g) \mapsto f+g(x) := f(x)+g(x)$$
$$\cdot \colon G \times G \to G$$
$$(f,g) \mapsto f \cdot g(x) := f(x)g(x)$$
$$\circ \colon G \times G \to G$$
$$(f,g) \mapsto f+g(x) := f(g(x))$$

Prove or disprove the following statements :
- $(G,+)$ is associative
- $(G,-)$ is commutative
- $(G,-)$ is associative
- $(G,\cdot)$ is associative
- $(G,\cdot)$ is commutative
- $(G,\circ)$ is commutative

(2) In a group $(G,\cdot)$ show that left and right cancellation holds, i.e.
- (left cancellation) if $x \cdot h = g \cdot h$ then $x = g$
- (right cancellation) if $h \cdot x = h \cdot g$ then $x = g$

(3) Let $(G,\cdot)$ be a group. Show that for any $g, h \in G$ the following equations have unique solutions :
- $x \cdot g = h$
- $g \cdot x = h$

(4) Let $(G,*)$ be a group. Show that G has a unique idempotent element, that is and element $x \in G$ so that $x^2 = x$.

(5) Show that if G is a group satisfying $g * g = e$ for all $g \in G$, then G is abelian.

(6) Show that if G is a group satisfying $g * g * g = e$ for all $g \in G$, then G is abelian.

(7) Decide whether the following subsets are subgroups of $G = GL(n, \mathbf{R})$:
- $H = \{M \in G \mid \det(M) = 2\}$
- $H = \{M \in G \mid \det(M) = \pm 1\}$
- $H = \{M \in G \mid M \text{ is diagonal}\}$
- $H = \{M \in G \mid M \text{ has no zeros on the diagonal}\}$

(8) Consider the set $G = C^0(\mathbf{R})$ as a group under pointwise multiplication :
- $H = \{f \in G \mid f(0) = 0\}$
- $H = \{f \in G \mid f(0) = 1\}$
- $H = \{f \in G \mid f(1) = 1\}$
- $H = \{f \in G \mid f \text{ is constant}\}$

(9) Let G be a group and H and K be subgroups. Show that
- $H \cap K$ is a subgroup, and
- $H \cup K$ is not necessarily a subgroup, by giving an example.