

MATH 504
EXERCISES 5

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Unless otherwise stated G and G' are groups and X is a non-empty set admitting an action of G .

- (1) Determine the Sylow p -subgroups of the following groups, for all primes p dividing $|G| = n$:
 - ▶ $G = A_4$
 - ▶ $G = \mathbf{Z}/12\mathbf{Z}$
 - ▶ $G = A_5$
 - ▶ $G = \mathfrak{S}_5$
 - ▶ $G = Q_8$
 - ▶ $G = \mathfrak{S}_3 \times \mathfrak{S}_3$
- (2) Find a Sylow 2-subgroup of \mathfrak{S}_4 .
- (3) Give two different examples of a Sylow 5-subgroup of \mathfrak{S}_6 , say H_1 and H_2 , and find an element $g \in \mathfrak{S}_6$ so that $H_1 = gH_2g^{-1}$.
- (4) Give two different examples of a Sylow 7-subgroup of \mathfrak{S}_7 , say H_1 and H_2 , and find an element $g \in \mathfrak{S}_7$ so that $H_1 = gH_2g^{-1}$.
- (5) Prove Cauchy's theorem using first Sylow theorem.