Université Galatasaray, Département de Mathématiques **Math 504 - Advanced Algebra** **Quiz 1,** *25/10/2021*		
Name & Surname:	**ID:**	$\sum$

1. Let G be a group. An *automorphism* of G is defined as a group isomorphism $\varphi\colon G \to G$.

i. Show that the set $\mathrm{Aut}(G) := \{\varphi\colon G \to G \,|\, \varphi \text{ is an isomorphism}\}$ is a group under composition.

ii. For any $g_o \in G$, define the map

$$\begin{aligned} \varphi_{g_o}\colon G &\to G \\ g &\mapsto g_o^{-1} g g_o \end{aligned}$$

is an element of $\mathrm{Aut}(G)$.

iii. Show that the set $\mathrm{Inn}(G) := \{\varphi_{g_o} \,|\, g_o \in G\}$ is a normal subgroup of $\mathrm{Aut}(G)$.

iv. Determine $\mathrm{Inn}(G)$ when G is an abelian group.