Université Galatasaray, Département de Mathématiques **Math 504 - Advanced Algebra** **Quiz 2,** *01/11/2021*		
Name & Surname:	**ID:**	$\sum$

1. Consider the set :

$$H = \left\{ \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \mid \theta \in \mathbb{R} \right\}$$

i. Show that H is a subgroup of $\mathrm{SL}(2,\mathbb{R})$. Is it normal?

i. Show that the map

$$\begin{aligned} \bullet &: H \times \mathbb{R}^2 \to \mathbb{R}^2 \\ (\gamma, (x,y)) &\mapsto \gamma \bullet (x,y) := \gamma \begin{pmatrix} x \\ y \end{pmatrix} \end{aligned}$$

defines an action of H on $\mathbb{R}^2$.

iii. Describe the set of orbits, that is describe the set $\mathbb{R}^2/H$. Hint: Show that the length of the vector $\gamma \bullet \begin{pmatrix} x \\ y \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$ are equal.