

Université Galatasaray, Département de Mathématiques

Math 504 - Advanced Algebra

Quiz 2, 08/11/2021

Name & Surname:

ID:

$\Sigma$

1. Let  $X$  be the polynomial ring in 4 independent variables with integral coefficients, that is  $X = \mathbf{Z}[x_1, x_2, x_3, x_4]$ . Consider the map :

$$\begin{aligned} \bullet : \mathfrak{S}_4 \times X &\rightarrow X \\ (\sigma, p(x_1, x_2, x_3, x_4)) &\mapsto \sigma \bullet p(x_1, x_2, x_3, x_4) := p(x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}, x_{\sigma(4)}) \end{aligned}$$

i. Show that this defines an action of  $\mathfrak{S}_4$  onto  $X$ .

ii. Compute the stabilizer of  $x_1 + x_2$  under this action.

iii. Find all polynomials that are equivalent to  $x_1 + x_2$ , i.e. find  $[x_1 + x_2]$ .

iv. Can you establish a bijection between the quotient  $X/\mathfrak{S}_4$  and  $\mathbf{Z}[x]$ ?