| Université Galatasaray, Département de Mathématiques |  |  |  |
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| Math 504-Advanced Algebra |  |  |  |
| Quiz 3, 22/11/2021 | ID: | $\sum$ |  |
| Name \& Surname: |  |  |  |

1. Let $G$ be a group of order 12. In this exercise, we will show that if the Sylow 3 subgroup of $G$ is not normal, then $G \cong A_{4}$.
i. Determine the number of possibilities of $n_{3}$.
ii. Let $\varphi$ denote the permutation representation associated with the action of $G$ on the set of Sylow 3 subgroups. Show that $\operatorname{ker}(\varphi)=\{e\}$.
iii. Determine the number of order 3 elements in $G$.
iv. Complete the proof by counting the number of subgroups of order 12 of $\mathfrak{S}_{4}$.
