Université Galatasaray, Département de Mathématiques **Math 504 - Advanced Algebra** **Quiz 3,** *06/12/2021*		
Name & Surname:	**ID:**	$\sum$

1. Let R be a commutative ring with identity. An element $r \in R$ is called nilpotent if for some $n \in \mathbf{Z}_{=}$ we have $r^n = 0$. Show that if $r \in R$ is nilpotent, then $1 + r$ is a unit in R.

2. Let R be a commutative ring with identity and $R[X]$ denote the ring of polynomials over R. That is :

$$R[X] = \{r(X) = r_o + r_1 X + \ldots + r_n X^n \,|\, r_i \in R, n \in \mathbf{Z}_{\geq 0}\}$$

i. Show that $r(X)$ is a unit in $R[X]$ if and only if $r(0)$ is a unit in R and $r_1, \ldots, r_n$ are nilpotent. Hint: Use induction for the if part while showing the nilpotency. Then use the previous exercise.

ii. Show that $r(X)$ is nilpotent if and only if each coefficient, that is $r_o, r_1, \ldots, r_n$ are nilpotent.