Université Galatasaray, Département de Mathématiques **Math 504 - Advanced Algebra** **Quiz 7,** *03/01/2022*		
Name & Surname:	**ID:**	$\sum$

1. Let M be an R-module and N an R-submodule of M; where R is a commutative ring with identity. Assume that M is finitely generated.

i. Show, by an example that N need not be a finitely generated R-module.

ii. Show, on the other hand, that N must be finitely generated if N can be realized as the kernel of a surjective R-module homomorphism $\varphi\colon M \to R^n$; for some $n \in \mathbf{Z}_+$.

Hint: Try to construct a short exact sequence $0 \to N \to M \to R^n \to 0$ which is split exact.

2. A question on tensor products.