Lecture Series II: Various arithmetic aspects on hypergeometric functions Hironori Shiga (Chiba Univ.)

Abstract In this series of talks we consider the following two themes:

(a) Consider the Schwarz map $D(x)$ for an Appell-Lauricella type diferential equation over the ground field of algebraic numbers. Describe the condition of an algebraic variable x to have an algebraic value $D(x)$ again. This is an analogy of the classical theorem by Theodor Schneider (Math. Ann. 1937).

(b) Consider a CM-field M that is embedded in some indefinite quaternion algebra over a totally real field. Describe the Hilbert class field of M by an adjunction of a special value of some modular function. It is contained in the frame work of the Shimura complex multiplication theory, and is a generalization of a typical case of "Kronecker Jugendtraume".

In the first week lectures (Part I), we show the over view, the outline and the statement of results. In the second week lectures (Part II), we speak about the details with introductory preparations of basics.

Part I (a) The Schwarz map of hypergeometric differential equations from the view point of period map.

(b) The Schwarz inverse map from the view point of the modular functions.

Part II (i) Introduction to the Hilbert class field and the complex multiplication.

(ii) Quaternion algebra and the arithmetic triangle group.

(iii) Theta representation of the modular function.

(iv) One visualized Shimura complex multiplication.

(v) Aspect from the theory of the $K3$ modular function.

Every talk is one-hour lecture.