

# The mini-symposium (Japanese Turkish Joint Geometry Meeting)

25-26 November 2013  
Galatasaray University, Istanbul, Turkey

## 1 Schedule

### 25 Nov.

09 : 30 – 10 : 20 **Serge Randriambololona** (Galatasaray University)  
10 : 20 – 10 : 30 Break  
10 : 30 – 11 : 20 **İsmail Sağlan** (Koc University)  
11 : 20 – 11 : 40 Break  
11 : 40 – 12 : 30 **Raimundas Vidunas** (National Kapodistrian University of Athens)  
12 : 30 – 14 : 00 Lunch  
14 : 00 – 14 : 50 **Kazim Büyükböduk** (Koc University)  
14 : 50 – 15 : 00 Break  
15 : 00 – 15 : 50 **Tadashi Ishibe** (University of Tokyo)  
15 : 50 – 16 : 00 Break  
16 : 00 – 16 : 50 **Jiro Sekiguchi** (Tokyo University of Agriculture and Technology)  
16 : 50 – 17 : 00 Break  
17 : 00 – 17 : 50 **Asli Deniz**

### 26 Nov.

09 : 30 – 10 : 20 **Jiro Sekiguchi** (Tokyo University of Agriculture and Technology) TBA.

## 2 Title & Abstract

### (1) Speaker: **Serge Randriambololona**

Title: The complex heritage of a real set (Joint work with J. Adamus and R. Shafikov)

Abstract:

Given a real analytic set  $X$  embedded in a complex analytic manifold  $Z$ , it is natural to try to measure how much of the complex structure of  $Z$  is inherited, locally at a point, by the real set  $X$ . One of the measure for the local "complexity" of  $X$  at the point  $p$  is the maximal possible dimension of a complex analytic germ contained in the germ of  $X$  at  $p$ . We will show that this notion of local dimension on  $X$  is well-behaved, giving rise to a filtration by closed semianalytic sets.

### (2) Speaker: **İsmail Sağlan**

Title: Triangulations of the Sphere After Thurston

Abstract:

In his paper 'Shapes of polyhedra and triangulations of the sphere', Thurston proved that all non-negatively curved triangulations of the sphere can be parametrized by a quotient of positive part of a hermitian lattice. We prove that same result is true for smaller families of non-negatively curved triangulations.

(3) Speaker: **Raimundas Vidunas**

Title: Differential relations for Belyi functions

Abstract:

Many Belyi functions give interesting pull-back transformations of Fuchsian differential equations. The pull-back transformations allow to compute the Belyi functions more efficiently, as they give additional relations for their coefficients. Genus 0 Belyi functions of degree 60 can be computed in a few minutes using the pull-back transformations. The implied differential relations between their polynomial components explain appearance of Chebyshev and Jacobi polynomials with Belyi functions. Those relations allow fast computations of Klein's pull-backs for algebraic hypergeometric or Heun functions.

(4) Speaker: **Kazim Büyükböyük**

Title: Beilinson-Kato elements and a conjecture of Mazur, Tate and Teitelbaum

Abstract:

In order to formulate a  $p$ -adic Birch and Swinnerton conjecture (BSD for short) for an elliptic curve  $E$ , Mazur, Tate and Teitelbaum (MTT) constructed a  $p$ -adic  $L$ -function attached to  $E$ . To understand its compatibility with the usual BSD, one needs to compare the order of vanishing of the  $p$ -adic  $L$ -function at  $s = 1$  to that of the Hasse-Weil  $L$ -function (where the latter is called the analytic rank of  $E$ ). When  $E$  has split multiplicative reduction mod  $p$ , MTT observed that the  $p$ -adic  $L$ -function always vanishes at  $s = 1$  and they conjectured that its order of zero is exactly one more than the analytic rank of  $E$ . In 1992, Greenberg and Stevens proved this conjecture when the analytic rank is zero. In this talk, I will explain a proof of the MTT conjecture when the analytic rank is one. Statistically speaking, this will complete the proof of MTT conjecture in almost all cases. The main ingredients for the proof are the Beilinson-Kato elements in the  $K_2$  of modular curves and a Gross-Zagier-style formula we prove for the  $p$ -adic height of the Beilinson-Kato elements.

(5) Speaker: **Tadashi Ishibe**

Title: On the conjugacy problem for non-Garside groups

Abstract:

Through a generalization of the theory of Artin groups, I would like to understand the elliptic Artin groups, which are the fundamental groups of the complement of discriminant divisors of the semi-versal deformation of the simply elliptic singularities  $\widetilde{E}_6$ ,  $\widetilde{E}_7$  and  $\widetilde{E}_8$ . Early in 70's the braid groups are generalized, by Brieskorn-Saito, to a wider class of groups, the fundamental groups of regular orbit spaces of finite reflection groups, which are called the Artin groups. The fundamental groups admit a special presentation, by which a certain monoid structure is naturally defined, which is called the Artin monoid. By showing a certain lemma for Artin monoid, we conclude that Artin monoid injects into the corresponding Artin group. Due to the injectivity, some decision problems for Artin groups is successfully solved. At the end of 90's, by referring to these technical framework in Artin group theory, the notion of Garside group, as a generalization of Artin groups, is defined as the group of fractions of a Garside monoid. In my opinion, the Garside theory is still far from complete to understand the elliptic Artin groups. In this talk, we will consider another generalization of the theory Artin groups. When we attempt to do so, we find difficulty in solving the conjugacy problems. We will talk about how to overcome that difficulty.

(6) Speaker: **Jiro Sekiguchi**

Title: A free divisor which gives an algebraic solution of Painlevé VI equation constructed by Hitchin.

Abstract:

Hitchin constructed some of algebraic solutions of Painlevé VI equation. My talk is focused on one of Hitchin's algebraic solutions. There are three Icosahedral invariants constructed by F. Klein whose degrees are 12, 20, 30. Let  $F(x_1, x_2, x_3) = x_3^3 + c_2(x_1, x_2)x_3 + c_3(x_1, x_2)$  be a weighted homogeneous polynomial of  $x_1, x_2, x_3$  of weight system  $(1, 2, 4)$ . Assume that  $c_2(z_1^{5/2}, z_2^5), c_3(z_1^{5/2}, z_2^5)$  are Icosahedral invariants of degree 20, 30 respectively, up to constant factors. Then  $F(x_1, x_2, x_3) = 0$  defines a free divisor. It is possible to construct a holonomic system of rank two with singularities along  $F = 0$ . This holonomic system is regarded as a family of ordinary differential equations of  $x_3$  with parameters  $x_1, x_2$ , in other words, the so called an isomonodromic deformation of ordinary differential equations and one obtains an algebraic solution of Painlevé VI equation. This algebraic solution coincides with that obtained by Hitchin.

(7) Speaker: **Asli Deniz**

Title: An Application of Hyperbolic Metric to Holomorphic Dynamics

Abstract:

In transcendental dynamics, dynamic rays are curves in the Julia set escaping to infinity under forward iterates. Understanding the landing behaviors of dynamic rays is essential in the study of the topological structure of the Julia set. In this talk, we present a landing theorem for periodic dynamic rays for transcendental entire maps which have bounded post singular sets. Our main tools are contraction principles in hyperbolic geometry, namely, Schwarz-Pick's Lemma.