

Arithmetic and Low Dimensional Hyperbolic Spaces

Galatasaray University, 27 June - 1 July 2016

MONDAY

- **10:00 – 11:00** Tomoyoshi Ibukiyama (Osaka)
- 11:00 Tea
- **11:30 – 12:30** Ekin Özman (Boğaziçi)
- 12:30 Lunch
- **15:00 – 16:00** Haluk Şengün (Sheffield)

TUESDAY

- **10:00 – 11:00** Hironori Shiga (Chiba)
- 11:00 Tea
- **11:30 – 12:30** Gabino González-Diez (Madrid)
- 12:30 Lunch
- **14:30 – 15:30** Hatice Boylan (Istanbul)
- 15:30 Tea
- **16:00 – 17:00** Vladimir Bavula (Sheffield)

WEDNESDAY

extra-curricular activity day: no talks

THURSDAY

joint day with the Workshop on Diffeology

- **10:00 – 11:00** Patrick Iglesias-Zemmour (Aix-Marseille)
- 11:00 Tea
- **11:30 – 12:30** Muhammed Uludağ (Galatasaray)
- 12:30 Lunch
- **15:00 – 16:00** Pierre Cartier (IHES)

FRIDAY

- **10:00 – 11:00** Gareth Jones (Southampton)
- 11:00 Tea
- **11:30 – 12:30** Robert Kucharczyk (ETH Zürich)
- 12:30 Lunch
- **15:00 – 16:00** İsmail Sağlam (Adana Bilim ve Teknoloji)

ABSTRACTS

MONDAY

TOMOYOSHI IBUKIYAMA:

Siegel modular forms and quaternion hermitian forms.

I would like to talk on automorphic forms of various \mathbb{Q} -forms of symplectic group of rank 2. We mainly focus on explicit dimension formulas of Siegel modular forms and automorphic forms of other \mathbb{Q} -forms (compact or non-compact non-split quaternion hermitian groups), some of which are related with hyperbolic space. This contains for example precise class number formulas of positive binary quaternion hermitian forms and some explicit conjectures on Langlands type correspondence. Some are my old results but some are new, containing also a survey on results done by other people.

EKİN ÖZMAN:

Solving Diophantine equations using modular curves.

Diophantine Equations are one of the central topics in number theory. We will introduce some methods to solve diophantine equations and see how modular curves will be applicable.

HALUK ŞENGÜN:

Number theory around arithmetic hyperbolic 3-manifolds.

We shall give an exposition of some number theoretic aspects of Bianchi manifolds, including connections with modular forms, elliptic curves and non-abelian class field theory.

TUESDAY

HIRONORI SHIGA:

Visualization of Shimura complex multiplication theorem via polygonal modular functions.

We show how to get the Shimura canonical model of a CM field coming from a quaternion algebra over a totally real field which corresponds to an arithmetic triangle group. As an application, we show explicit examples of the Hilbert class fields of a CM field of higher degree, also.

GABINO GONZÁLEZ-DIEZ:

Arithmeticity of complex surfaces and uniformization.

In this talk I shall attempt to show that whether or not a complex surface S of general type is arithmetic depends on whether or not it contains a Zariski open subset $U \subset S$ whose universal cover is isomorphic to the graph of a holomorphic motion of a certain kind. This characterization of arithmeticity in terms of universal covers is very much in contrast with the corresponding situation in complex dimension one, where the universal cover is always the unit disk. This is joint work with Sebastián Reyes-Carocca.

HATICE BOYLAN:

Representations of SL_2 over rings of integers of local fields, and over arithmetic Dedekind domains.

In various arithmetic-geometric applications and in the theory of automorphic forms there are open problems whose answer can be reduced to a question about finite dimensional representations of $SL(2, O)$, where O is a maximal order in a number field or, more generally, an arithmetic Dedekind domain. It is amazing that even natural questions like for the group of linear characters of such groups did until recently not have a satisfactory answer.

In the present talk we describe recent progress in the theory of finite dimensional representations of $SL(2, O)$ for a fairly large class of rings O comprising the rings of integers of local fields and arithmetic Dedekind domains. Amongst other things we describe all linear characters of these groups $SL(2, O)$. We show how to use the general theory of Weil representations to construct finite dimensional representations of these $SL(2, O)$. We indicate why these so constructed families of representations possibly contain all finite dimensional representations with finite image of these $SL(2, O)$ (except for certain O). We finish with some open questions concerning the classification of the central extensions of these $SL(2, O)$ by the cyclic group of order 2.

VLADIMIR BAVULA:

New criteria for a ring to have a semisimple left quotient ring.

Goldie's Theorem (1960), which is one of the most important results in Ring Theory, is a criterion for a ring to have a semisimple left quotient ring. The aim of the paper is to give four new criteria (using a completely different approach and new ideas). The first one is based on the recent fact that for an arbitrary ring R the set M of maximal left denominator sets of R is a non-empty set.

THURSDAY**PATRICK IGLESIAS-ZEMMOUR:**

Arithmetic and diffeology.

I will talk about a few questions where diffeology and arithmetics mix:

- (1) The first example is the condition of isomorphy between irrational tori $T_\alpha = T^2/\Delta_\alpha$, where Δ_α is the line of slope $\alpha \in \mathbf{R} - \mathbf{Q}$. We will see that this is defined by the action of $GL(2, \mathbf{Z})$ on the 1-dimensional projective space.
- (2) Next, we'll see that the group of components $\pi_0(\text{Diff}(T_\alpha))$ is different when α is quadratic or not. More generally, we'll use the Dirichlet's unit theorem to compute the $\pi_0 - \text{Diff}$ of the space of leaves of the linear 1-codimension irrational foliations on a n -dimensional torus.
- (3) Eventually we shall see that the group of classes of $(\mathbf{R}, +)$ -principal bundles on T_α depends on the diophantine property of α .

MUHAMMED ULUDAĞ:

The outer automorphism of $PGL(2, \mathbb{Z})$ and the induced 'modular' involution of the real line. (joint work with Hakan Ayrál)

We study the involution of the real line induced by the outer automorphism of the extended modular group $PGL(2, \mathbb{Z})$. This modular involution is discontinuous at rationals but satisfies a surprising collection of functional equations. It preserves the set of real quadratic irrationalities mapping them in a highly non-obvious way to

each other. It commutes with the Galois action on the set of real quadratic irrationals and establishes a bijection between the square roots of positive rationals and the units of negative norm in quadratic number fields. Moreover, it induces an involution of the units of positive norm.

More generally, it preserves setwise the orbits of the modular group on the real line, thereby inducing an involution of the moduli space of real rank-two lattices. It induces a duality of Beatty partitions of the set of positive integers. It also induces a subtle symmetry of Lebesgue's measure.

This involution conjugates the Gauss continued fraction map to an intermittent dynamical system on the unit interval with an infinite invariant measure. The transfer operator & functional equation naturally associated to this dynamical system is closely related to the Mayer transfer operator & Lewis functional equation. In this context it defines a certain parity between the Riemann zeta function and the 'Fibonacci zeta'.

We give a description of this involution as the boundary action of a certain automorphism of the Farey tree. Its derivative exists and vanishes almost everywhere. It is conjectured that algebraic numbers of degree at least three are mapped to transcendental numbers under this involution.

PIERRE CARTIER:

On the analogy between jet spaces and Witt vectors : a categorical approach. (In collaboration with A. Joyal)

FRIDAY

GARETH JONES:

Regular dessins with a given automorphism group.

I will discuss the set of all regular dessins with a given automorphism group, and in particular how these dessins may be enumerated, and how they are acted on by two particular groups: the absolute Galois group and the outer automorphism group of the free group of rank 2.

ROBERT KUCHARCZYK:

Modular embeddings and dessins d'enfants.

In this talk I will explain how the use of modular embeddings enables us to find a large class of dessins d'enfants embedded in Shimura varieties and thereby determine their Galois conjugates completely.

İSMAİL SAĞLAM:

Complete Euclidean cone metrics on punctured surfaces.

We prove that complete Euclidean cone metrics on punctured surfaces can be triangulated naturally. We derive Gauss-Bonnet Formula for these surfaces. We also prove existence of length minimizing geodesics in free homotopy classes of paths on such a surface.