JA2019 - Abstracts

1-5 July 2019

Speaker: Emre Alkan
Title: Number of shifted primes as k-free integers

Abstract: Investigating the multiplicative structure of shifted prime numbers is an interesting topic in analytic number theory and such numbers have been studied from various perspectives in the literature. In this talk, we will explore this topic from another viewpoint. Historically, improving on the works of Estermann, Page and Walfisz, Mirsky obtained an asymptotic formula for the number of representations of an integer as the sum of a prime number and a k-free integer. Recently, Languasco sharpened the error term in Mirsky's formula by assuming the nonexistence of Siegel zeros. In the same paper, Mirsky also gave an asymptotic formula for the number of prime numbers p such that p+a is k-free. Inspired by the contribution of Languasco, we present variants of Mirsky's formula for two consecutive shifts of prime numbers that are simultaneously k-free. Specifically, assuming that there are no Siegel zeros, we obtain an asymptotic formula, with a sharper error term, for the number of prime numbers p such that p-1 and p-2 are both k-free. We discuss further improvements of the error term by assuming weaker versions of the Generalized Riemann hypothesis for character L-functions. Time permitting, we will mention unconditional results that are sensitive to the values at shifted prime numbers of a family of arithmetic functions with mild growth conditions and discuss other consequences of our method.

Speaker: Kazım Büyükboduk
Title: p-adic Gross-Zagier formula at critical slope and a conjecture of Perrin-Riou

Abstract: I will report on joint work with R. Pollack and S. Sasaki, where we prove a p-adic Gross-Zagier formula for critical slope p-adic L-functions. Besides the strategy for our proof, I will illustrate a number of applications. The first is the proof of a conjecture of Perrin-Riou, which predicts an explicit construction of the generator of the Mordell-Weil group in terms of p-adic L-values when the analytic rank is one. The second is towards a Birch and Swinnerton-Dyer formula when the analytic rank is one, yielding an improvement of the recent results of Jetchev-Skinner-Wan in this context (and simplifying their

proof).

Speaker: Ekin Özman
Title: Modularity, Rational Points and Diophantine Equations

Abstract: Understanding solutions of Diophantine equations over rationals or more generally over any number field is one of the main problems of number theory. By the help of the modular techniques used in the proof of Fermat's Last Theorem and its generalizations, it is possible to solve other Diophantine equations too. Understanding rational points on the twists of the classical modular curve or quadratic points on it play a central role in this approach. In this talk, I will survey results in these directions including some recent results about quadratic points on the classical modular curve. This is joint work with Samir Siksek.

Speaker: Nikos Frantzikinakis
Title: The Möbius disjointness conjecture of Sarnak for ergodic weights

Abstract: The Möbius function is a multiplicative function which encodes important information related to distributional properties of the prime numbers. It is widely believed that its non-zero values fluctuate between plus and minus one in such a random way that causes non-correlation with any "reasonable" sequence of complex numbers. One conjecture in this direction, formulated by Sarnak, states that the Möbius function does not correlate with any bounded deterministic sequence, meaning, any sequence that is produced by a continuous function evaluated along the orbit of a point in a zero entropy topological dynamical system. I will describe the proof of the logarithmically averaged variant of this conjecture for a wide class of dynamical systems, which includes all uniquely ergodic ones. Our approach is to study structural properties of measure preserving systems naturally associated with the Möbius function. I will explain how these structural results are obtained using a combination of tools from ergodic theory and analytic number theory, and how we use them for our purposes.

This is joint work with Bernard Host.

Speaker: Javier Fresan
Title: From Feynman integrals to Kloosterman sums

Abstract: Certain Feynman integrals in two-dimensional quantum field theory are given by moments of the Bessel functions. In an attempt to get a grasp of their properties, Broadhurst and Roberts explored the analogy with symmetric power moments of Kloosterman sums over finite fields. I will prove that the L-functions assembling them arise from automorphic motives over the rational numbers, and therefore extend meromorphically to the complex plane and satisfy a functional equation. I will then identify the periods of the motives in question with the Bessel moments we started with and explain how Poincaré

duality produces quadratic relations among them. The talk is based on joint work with Claude Sabbah and Jeng-Daw Yu.

Speaker: Eva Bayer-Fluckiger
Title: Isometries of lattices

Abstract: In a joint work with Lenny Taelman, we characterize the irreducible polynomials that occur as a characteristic polynomial of an isometry of an even, unimodular lattice with given signature, answering a question of Gross and McMullen. It turns out that the criteria are local ones, and that in the case of an irreducible polynomial, one has a local-global principle. This is no longer true for reducible polynomials. The aim of the talk is to describe these results, give to a criterion for the local-global principle to hold, as well as to survey some earlier results and applications.

Speaker: Samir Siksek
Title: On the asymptotic Fermat conjecture

Abstract: The asymptotic Fermat conjecture states that for a number field K there is a constant B_K such that for primes $p \geq B_K$ the only K-rational points on the Fermat curve $X^p + Y^p + Z^p = 0$ are the obvious ones. In this talk we survey joint work with Nuno Freitas, Alain Kraus and Haluk Sengun, on the asymptotic Fermat conjecture. In particular we prove AFC for family of number fields $K = \mathbb{Q}(\zeta_{2^r})^+$.

Speaker: Charlotte Hardouin
Title: Walks, difference equations and elliptic surfaces

Abstract:A walk in the quarter plane is a path between integral points with a prescribed set of directions that is confined in the quarter plane. In the recent years, the enumeration of such walks has attracted the attention of many authors in combinatorics and probability. The complexity of their enumeration is encoded in the algebraic nature of their associated generating series. The main questions are: are these series algebraic, holonomic (solutions of linear differential equations) or differentially algebraic (solutions of algebraic differential equations)?

In this talk, we will show how the algebraic nature of the generating series can be approached via the study of a discrete functional equation over a curve E of genus zero or one over a function field and the Galois theory of difference equations. In the genus zero case, the functional equation corresponds to a so called q-difference equation and the generating series is differentially transcendental. In genus one, the dynamic of the functional equation is the addition by a given point P of the elliptic curve E. If the point P is torsion then the generating series is holonomic. When P is non torsion, the nature of the generating series is captured by the linear dependence relations of certain prescribed points in the Mordell-Weil lattice of an elliptic surface.

This work combines several collaborations with T. Dreyfus (Irma, Strasbourg), J. Roques (Institut Fourier, Grenoble) and M.F. Singer (NCSU, Raleigh).

Speaker: Jennifer Balakrishnan
Title: Rational points on the cursed curve

Abstract: The split Cartan modular curve of level 13, also known as the "cursed curve," is a genus 3 curve defined over the rationals. By Faltings' proof of Mordell's conjecture, we know that it has finitely many rational points. However, Faltings' proof does not give an algorithm for finding these points. We discuss how to determine rational points on this curve using "quadratic Chabauty," part of Kim's nonabelian Chabauty program. This is joint work with Netan Dogra, Steffen Mueller, Jan Tuitman, and Jan Vonk.

Speaker: Avi Wigderson
Title: Randomness

Abstract: Is the universe inherently deterministic or probabilistic? Perhaps more importantly - can we tell the difference between the two?

Humanity has pondered the meaning and utility of randomness for millennia. There is a remarkable variety of ways in which we utilize perfect coin tosses to our advantage: in statistics, cryptography, game theory, algorithms, gambling... Indeed, randomness seems indispensable! Which of these applications survive if the universe had no (accessible) randomness in it at all? Which of them survive if only poor quality randomness is available, e.g. that arises from somewhat "unpredictable" phenomena like the weather or the stock market?

A computational theory of randomness, developed in the past several decades, reveals (perhaps counter-intuitively) that very little is lost in such deterministic or weakly random worlds. In the talk I'll explain the main ideas and results of this theory, notions of pseudo-randomness, and connections to computational intractability.

It is interesting that Number Theory played an important role throughout this development. It supplied problems whose algorithmic solution make randomness seem powerful, problems for which randomness can be eliminated from such solutions, and problems where the power of randomness remains a major challenge for computational complexity theorists and mathematicians. I will use these problems (and others) to demonstrate aspects of this theory.