

Mathematics and Schizophrina

Analogy pervades all our thinking, our everyday speech and our trivial conclusions as well as artistic ways of expression and the highest scientific achievements.

– George Polya

A mathematician is a person who can find analogies between theorems; a better mathematician is one who can see analogies between proofs and the best mathematician can notice analogies between theories. One can imagine that the ultimate mathematician is one who can see analogies between analogies.

– Stefan Banach

As students of science we are often told to question even the most basic and foundational concepts in mathematics and in science, and we also tell our pupils to do so. However, in practice we tend to relegate such foundational inquiries to more fundamental disciplines, such as the mathematical logic, history of mathematics or even philosophy. Since we want to see ourselves manipulating more sophisticated and serious questions, formulated in a much higher-level language, we are content to know that there is someone “down there” wasting his time by this childish foundational analysis.

However, there will always remain a doubt that a foundational stone have been misplaced or have been forgotten to be put in its place, and that this misplaced stone is generating, via some mysterious mechanism, some structural phenomena on “higher levels” that we contemplate with admiration but unable to grasp without knowing why and how that stone have been laid so.

In order to protect ourselves from the apparent naivety of this foundational inquiry, we choose to live with this doubt, and we end up believing that if there were such phenomena having their roots in foundational level, they could be detected and dealt with on a higher level, where they appear much less naive. Also, a foundational change might require the modification of many concepts and breaking centuries old mathematical traditions, so it is much more practical, for sociological reasons, to face with them on higher ground.

As a consequence of our unwillingness to undertake this inquiry, on the foundational level, we comfort ourselves by explaining some phenomena as analogies. Some of us even became mystics by accepting the existence of general phenomena that becomes manifest in individual theories, so called ‘avatars’, et cetera. When this similitudes are laid by force into the theory then they are baptized and eulogized as metaphors.

When astronomers observe some phenomenon in some distant star, for example its spectrum, which turns out to be similar to what we observe in our own sun, perhaps up to a shift, they don’t give their final verdict on the issue by stating that the phenomenon in question is an “analogy”. They persevere to find an explanation, based on the firm belief in

the common origin of these phenomena. As a consequence of which they end up explaining the cause of the shift as well. Similarly, when a historian of nature observes that the whale fins hide five fingers inside, just as many fingers as he has in his hand, he won't conclude that there are some bizarre "analogies" in nature. The explanation is evident – and any such structural similarities which can not be explained by common origins are understood as being a result of their common purpose. How ironically perfect and flawless their logic is!

But in mathematics too, the first dominion of the queen of logic, there can be no such conceptual "non-localities", islands of mathematical phenomena, which are totally isolated one from another but becomes manifest in some strange allegoric ways. The unity of our discipline and our consciousness demands that such similitudes must be understood via a common conceptual root. (At this point I must remind you that understanding is an active process which requires the construction of necessary tools to reach its end.) This is because ideas are being (or rather 'must be') generated through a certain birth process. Perhaps we perceive that this conceptual root, this texture of our mathematical universe, is so primitive that it can no be constructed nor conceptualized nor communicated. Also, this texture is so fine that its structure will not affect in any significant way the higher phenomena that we try to understand by modeling on this texture which is thus viewed as a palette on which we paint our thoughts. We need not to bother about the structure of this palette as long as it conveys our ideas. It is like a tabula rasa, indeed a direct emotional abstraction of the blank sheet of paper, our starting point of day's work.

I can not say that this point of view is entirely unjustified. However, every time we observe some new phenomenon, every time we discover some analogy, if we are looking at things with a sufficiently high resolution we are in fact faced with that texture. By painting on this palette, all we do is, in fact, to qualify and quantify that texture. Analogies are toys of the genius, who use them to quickly jump from one idea to the other, often themselves unable to grasp how their intuition have teleported them.

If you imagine yourself as constructing the tip of a huge crystal tower, or as giving combat on the most recent front of scientific knowledge, you are in an illusion and afar from the mathematical truth. This is so; even if you are able to successfully produce some interesting theorems and convince some others to follow you. Since, irremediably, the search of mathematical truth requires an eternal return to a study of its first principles. Thus the processes by which mathematical objects are generated must become our subject of inquiry in an endless cycle, as this is the *perpetuum mobile* machine which generates new ideas.

What is regretful is that, this foundational inquiry lies at the roots every significant scientific development, and especially when this development is revolutionary and brings forth a change of paradigm and also of language, anything happening before that change is rendered meaningless, together with an important part of the foundational inquiry itself which gave birth to it. A new world is created, with its language that becomes quickly standard, in terms of which the myriad open questions of the new theory are formulated. What remains of the foundational inquiry becomes anecdotal accounts of the pre-revolutionary period. New recruits busy with their work inside the new paradigm regard these as naive inquiries on the way to the discovery of the paradigm.. And so the metaphor pervades.

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