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BIRKHÖUSER



Arrangements, Local Systems and Singularities

CIMPA Summer School,
Galatasaray University, Istanbul, 2007

Fouad El Zein
Alexander I. Suciuc
Meral Tosun
A. Muhammed Uludağ
Sergey Yuzvinsky
Editors



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H. Bass

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Birkhäuser
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Editors:

Fouad El Zein
Institute of Mathematics
Case 7012, 2 place Jussieu
75251 Paris Cedex 05
France
e-mail: elzein@math.jussieu.fr

and

Center for Advanced Mathematical
Sciences (CAMS)
American University of Beirut
P.O. Box 11-236
Beirut
Lebanon

Alexandru I. Suciú
Northeastern University
Department of Mathematics
360 Huntington Avenue
Boston, MA 02115
USA
e-mail: a.suciú@neu.edu

Meral Tosun
Galatasaray University
Mathematics Department
Ciragan Cad. No 36
Besiktas, 34357 Istanbul
Turkey
e-mail: mrltosun@gmail.com

A. Muhammed Uludağ
Galatasaray University
Mathematics Department
Ciragan Cad. No 36
Besiktas, 34357 Istanbul
Turkey
e-mail: muhammed.uludag@gmail.com

Sergey Yuzvinsky
Department of Mathematics
University of Oregon
Eugene, OR 97405
USA
e-mail: yuz@uoregon.edu

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Preface

This volume comprises a set of lecture notes from the CIMPA Summer School, *Arrangements and Local systems and Singularities*, held at Galatasaray University, Istanbul, during June 11-22, 2007. The school was attended by 68 mathematicians, 35 of them from 19 countries outside Turkey. The Summer School was made up of eleven short courses and five seminars presented by an outstanding group of lecturers who covered a wide range of topics related to the concepts of arrangements, local systems and singularities. The list of lectures of the workshop appears below. Most members of the audience were graduate students or young researchers and the primary purpose of the school was to introduce them, in particular those from developing countries, to the many advances and opportunities in this widely applicable field.

Historically, research on arrangements of hyperplanes, starting with the work of V.I. Arnold and E. Brieskorn and later Peter Orlik, was presented at meetings devoted mainly to discussions of singularities. Some of the articles in this volume demonstrate that the interaction between these two subjects is ongoing and particularly productive. Beyond this, the volume is intended for a large audience in pure mathematics, including researchers and graduate students working in algebraic geometry, singularity theory, topology and related fields. The reader will find in the Problem Section at the end of the volume a variety of open problems proposed by the lecturers at the end of the School's sessions as directions ripe for further study and development.

We would like to thank the *Centre des Mathématiques Pures et Appliquées (CIMPA)* for sponsoring the School and Professor Richard Grin for organizational help. We would also like to thank the *Scientific and Technological Research Council of Turkey (TUBITAK)* for their financial help which made it possible to invite 16 young mathematicians from Turkey as well as some lecturers from abroad. We were able to support participants from across the region, thanks to the generous financial help provided by the *International Center for Theoretical Physics (ICTP)* and the *International Mathematical Union (IMU)*.

During the long preparatory process and also during the school, Gülay Kaya, Ayşe Altıntaş and Celal Cem Sarioğlu contributed at various levels to the organization. We are grateful to them.

F. El Zein, A. Suciu, M. Tosun, A.M. Uludağ and S. Yuzvinsky

The fourth named editor was supported by Tübitak/Kariyer 103T136 and Tuba/Gebip during the summer school and during the preparation of this volume. The third and fourth named authors were supported by the Galatasaray University Research Fund during the preparation of this volume.

List of Lectures of the School

T. Abe

Addition-deletion theorems for multiarrangements and graphic deformation of braids.

D. Cohen

Fundamental groups of complements of hyperplane arrangements.

G. Denham

Homological algebra of hyperplane arrangements.

F. El Zein

Local systems, constructible sheaves and Geometry.

M. Falk *Geometry and combinatorics of resonance weights.*

G. Kaya

Moduli of surfaces admitting non-smooth genus two fibrations over elliptic curves.

A.U.O. Kisisel

Toric varieties and the diagonal properties.

Lê D.T.

Simple singularities and simple Lie algebras.

D. Matei

Artin groups, Bestvina-Brady groups and arrangements of hypersurfaces.

D. Mond

(1) *Introduction to singularities of mappings*

(2) *Conservation of multiplicity. Cohen-Macaulay rings. Free divisors (from groups).*

M. Oka

Introduction to plane curve singularities.

H. Terao

(1) *The characteristic polynomial of a multiarrangement*

(2) *Periodicity of arrangements with integral coefficients modulo positive integers.*

M. Yoshida

Hyperbolic Schwarz map for the hypergeometric differential equation.

M. Yoshinaga

Minimal CW decomposition of the complement of hyperplane arrangements.

S. Yuzvinsky

Orlik-Solomon algebras of hyperplane arrangements.

List of Participants

Name	Institution
Aase Feragen	University of Helsinki
Abdigappur Narmanov	National University of Uzbekistan
Ayberk Zeytin	Middle East Technical University
Ayşe Altıntaş	University of Warwick
Ayşegül Özgüner	Bilkent University
Barbu Berceanu	IMAR Bucharest
Bayat Farshad Merrikh	Sharif University of Technology
Bedia Akyar Møller	Dokuz Eylül University
Begüm Uçar	Yıldız Technical University
Bülent Tosun	Middle East Technical University
Celal Cem Sarioğlu	Dokuz Eylül University
Daniel Cohen	Louisiana State University
Daniel Matei	IMAR Bucharest
David Mond	University of Warwick
Duygu Irmak	Yıldız Technical University
Emanuele Delucchi	Università Di Pisa
Emel Bilgin	İstanbul Technical University
Emel Coşkun	İstanbul Technical University
Fatma Şengüler	İstanbul Technical University
Fouad El Zein	Université de Nantes
Graham Denham	University of Western Ontario
Guljakhan Kaypnazarova	National University of Uzbekistan
Gülây Kaya	Galatasaray University
Gülçin Gökçe	Ondokuz Mayıs University
Hakan Güntürkün	Middle East Technical University
Hani Shaker	Abdus Salam School of Mathematical Sciences GCU
Hiraoki Terao	Hokkaido University
Javier Bobadilla	CSIC
Kaan Esin	İstanbul Technical University
Khurram Shabbir	Abdus Salam School of Mathematical Sciences GCU

Kouider Djerfi	Centre de Universitaire de Saida
Kürşat Hakan Oral	Yıldız Technical University
Lê Dững Tráng	ICTP
Mamuka Shubladze	Tbilisi State University
Maria Pe Pereira	Universidad Complutense de Madrid
Masaaki Yoshida	Kyushu University
Masahiko Yoshinaga	ICTP
Meral Tosun	Galatasaray University
Mesut Arslandok	Yıldız Technical University
Michael Falk	Northern Arizona University
Mikhail Mazin	University of Toronto
Mohammed Salim Jbara	University of Baghdad
Mohan Bhupal	Middle East Technical University
Muhammed Uludağ	Galatasaray University
Mustafa Topkara	Middle East Technical University
Mutsuo Oka	Universite de Tokyo Science
Münevver Çelik	Middle East Technical University
Ömer Faruk Koç	Yıldız Technical University
Özer Öztürk	Middle East Technical University
Özgür Kişisel	Middle East Technical University
Paul Cadman	University of Warwick
Pınar Mete	Bahkesir University
Richard Grin	CIMPA
Sabri Kaan Gürbüzler	Dokuz Eylül University
Selma Altınok	Adnan Menderes University
Sergei Yuzvinsky	University of Oregon
Shaheen Nazir	Abdus Salam School of Mathematical Sciences GCU
Shiv Datt Kumar	Motilal Nehru National Institute of Technology
Sultan Erdoğan	Bilkent University
Takeshi Sasaki	Kobe University
Takuro Abe	Hokkaido University
Tevfik Şahin	Ondokuz Mayıs University
Thi Anh Thu Dinh	Laboratoire Ja Dieudonne
Yasemin Güllük	Yıldız Technical University