Extension of Galois groups by solvable groups, and application to fundamental groups of curves

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first talk (first three sections)

title : Pro-solvable fundamental group of an affine curve

abstract : We will start with a quick survey of classical results on fundamental groups of curves over an algebraically closed field. We will then explain a natural cohomological method, due to Serre, that allows to extend the Galois group of a given cover by an abelian group. As an application, we give an algebraic proof of (a weak form of) the theorem giving the structure of the largest (pro-solvable) p'-quotient of the fundamental group of an affine curve.

references : [SGA03, Sza09, BE08]

second talk (next two sections)

title : Grothendieck-Ogg-Shafarevich formula, and an application

abstract : We will give details about the classical Grothendieck-Ogg-Shafarevich formula, including a sketch of a proof, and then describe an application, namely Serre's proof of Abhyankar's conjecture for solvable groups on $\mathbb{A}^1$.

references : [SGA77, Ray95, Ser90]

third talk (last two sections)

title : Fundamental groups of complete curves in positive characteristic

abstract : We will give a relative (or equivariant) version of Shafarevich's theorem describing the largest p-quotient of the fundamental group of a complete curve in positive characteristic p, and then conclude with some open problems.

references : [Nak85, PS00, Bor04]

5 Abhyankar's conjecture for solvable groups on $\mathbb{A}^1$

5.1 The Rev_p property

5.2 The case $l \neq p$

5.3 The case $l = p$

6 Fundamental groups of complete curves in positive characteristic

6.1 p-cohomological dimension

6.2 Semi-simple differentials

6.3 Nakajima's structure theorem

6.4 Extension of a Galois cover by p-group

6.5 Sketch of a proof

7 Open problems

7.1 Abhyankar's conjecture for solvable groups on an affine curve

7.2 Largest pro-solvable p'-quotient of the fundamental group of a complete curve

Références

[BE08] Niels Borne and Michel Emsalem. Note sur la détermination algébrique du groupe fondamental pro-résoluble d'une courbe affine. *J. Algebra*, 320(6) :2615–2623, 2008.

[Bor04] Niels Borne. A relative Shafarevich theorem. *Math. Z.*, 248(2) :351–367, 2004.

[Nak85] Shōichi Nakajima. Equivariant form of the Deuring-Šafarevič formula for Hasse-Witt invariants. *Math. Z.*, 190(4) :559–566, 1985.

[PS00] Amílcar Pacheco and Katherine F. Stevenson. Finite quotients of the algebraic fundamental group of projective curves in positive characteristic. *Pacific J. Math.*, 192(1) :143–158, 2000.

[Ray95] Michel Raynaud. Caractéristique d'Euler-Poincaré d'un faisceau et cohomologie des variétés abéliennes. In *Séminaire Bourbaki, Vol. 9*, pages Exp. No. 286, 129–147. Soc. Math. France, Paris, 1995.

[Ser90]   Jean-Pierre Serre. Construction de revêtements étales de la droite affine en caractéristique p. *C. R. Acad. Sci. Paris Sér. I Math.*, 311(6) :341–346, 1990.

[SGA77]   *Cohomologie l-adique et fonctions L*. Lecture Notes in Mathematics, Vol. 589. Springer-Verlag, Berlin, 1977. Séminaire de Géometrie Algébrique du Bois-Marie 1965–1966 (SGA 5), Edité par Luc Illusie.

[SGA03]   *Revêtements étales et groupe fondamental (SGA 1)*. Documents Mathématiques (Paris) [Mathematical Documents (Paris)], 3. Société Mathématique de France, Paris, 2003. Séminaire de géométrie algébrique du Bois Marie 1960–61. [Algebraic Geometry Seminar of Bois Marie 1960-61], Directed by A. Grothendieck, With two papers by M. Raynaud, Updated and annotated reprint of the 1971 original [Lecture Notes in Math., 224, Springer, Berlin; MR0354651 (50 #7129)].

[Sza09]   Tamás Szamuely. *Galois Groups and Fundamental Groups*, volume 117 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2009.